Title

S-(convex/concave) orderings on an arbitrary grid with applications to finance and actuarial science

Abstract

It is well established that the theory of stochastic orderings has a considerable interest in probability and actuarial sciences, for theoretical and practical purposes (see, e.g., [7], [5] and [6]). For instance, it can be used to compare complex models with more tractable ones which are “riskier”, leading thus to more conservative decisions.

Traditionally, stochastic order relations are used to compare random variables which take on values in $\mathbb{R}$, $\mathbb{R}^+$ or an entire interval. However, in actuarial science, many random variables are discrete by nature (counts for instance) and it is a great need to dispose of stochastic orderings to compare such arithmetic random variables. The basic situation is when all outcomes lie in a finite set $D_n$ of $(n+1)$ evenly-spaced points, with minimum $e_0$ and separation parameter $h > 0$ say, i.e. $D_n = \{e_0 + ih : i = 0, \ldots, n\}$.

A remarkable class of integral stochastic orderings investigated by [2] is the class of the discrete $s$-convex orderings among arithmetic random variables valued in $D_n$. These orderings are defined using the traditional $s$-th order forward difference operator and have been studied in details in [2], [3] and [1].

It is worth mentioning that these orderings have been generalized by [4], using the concept of divided difference operator on $\mathbb{R}$ substituted for the operator of forward differences, to compare any pair of discrete random variables valued in an arbitrary (rather than equidistant) ordered finite grid of nonnegative points, denoted by $E_n = \{e_0, \ldots, e_n\}$ say. These orderings are of direct interest in various fields of applications, especially for problems of risky decision making, portfolio selection and of insurance premium evaluation.

It is important to consider the $s$-(convex/concave) orderings (and their extrema) on an arbitrary grid $E_n$ rather than on a “equidistant” grid $D_n$ or even on continuous sets of reals $[a, b]$ for the following reason: [4] considers how an extension/restriction of the grid $E_n$ can affect the stochastic orderings of (convex/concave)-type. Clearly a restriction to a subset of the grid does not affect the $s$-(convex/concave) orderings. It is also the case when $s = 1$ and $s = 2$. However they pointed out that for $s \geq 3$ (case that is more and more encountered in research), the location of the additional point that is inserted in the grid is a crucial factor and two random variables ordered on $E_n$ are no longer necessarily ordered on a larger subset $E^* \subset E_n$.

These results lead to the following observation: it is always safer to consider random variables valued in a smaller set of outcomes rather than in a larger one. For example, in the context of decision analysis, if the decision-maker’s preferences are determined by some stochastic ordering of (convex/concave)-type and that there is an important variability in the supports of outcomes for the different random alternatives, we make the following observations. First, when comparing two alternatives, it is safer to consider them valued in a smaller set of outcomes rather than in a larger one, because any such comparison can be
extended to a larger set but not reciprocally. Moreover, outlier outcomes which are located outside of the central outcomes set, do not affect s-(convex/concave) orderings, while outliers inside the set can do it.

In this paper, our purpose is thus to obtain the minimum and maximum in the stochastic s-convex sense for random variables valued in $\mathcal{E}_n$. To begin with, we give some basic characterizations and properties about the orderings. In particular, we recall that only random variables with identical first $s-1$ moments can be compared in the s-convex sense. This leads us to examine the associated moment space $\mathcal{D}_s (\mathcal{E}_n; \mu_1, \ldots, \mu_{s-1})$ which contains all random variables $X$ in $\mathcal{E}_n$ such that the first $s-1$ moments are fixed to $\mathbb{E}X^k = \mu_k$, $k = 1, \ldots, s-1$, where $s$ is a prescribed nonnegative integer. Then we construct the extrema in this space using a methodology extending the one of [1]. Finally, we give some simple actuarial and financial applications where the use of the studied discrete orderings and their extrema is quite useful.

Keywords: Discrete stochastic orderings, arbitrary grid, cut-criterion, stochastic extrema

References


Coordinates

Cindy COURTOIS
Institut des Sciences Actuarielles, Université catholique de Louvain
Rue des Wallons 6, B-1348 Louvain-la-Neuve, Belgium
E-mail address: courtois@stat.ucl.ac.be
Phone: +32 10 47.93.84
Fax: +32 10 47.94.32
URL: http://www.actu.ucl.ac.be/staff/courtois/ccourtois.html