Optimal joint survival reinsurance: an efficient frontier approach

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Abstract

Several approaches to optimal reinsurance have been attempted in the actuarial literature, based on risk theory, economic game theory and stochastic dynamic control. Examples of research in each of these directions are the papers by Dickson and Waters (1996, 1997), Centeno (1991, 1997), Andersen (2000), Krvavych (2001), by Aase (2002), Suijs, Borm and De Waegenaere (1998), and by Schmidli (2001, 2002), Hipp and Vogt (2001), Taksar and Markussen (2003). A common feature of most of the quoted works is that optimality is considered with respect to the interest of solely the direct insurer, minimizing his (approximated) ruin probability, under the classical assumptions of linearity of the premium income function and independent, identically distributed claim severities.

Recently, a reinsurance optimality model, which takes into account the interests of both parties to a reinsurance contract, the cedent and the reinsurer, has been proposed by Ignatov, Kaishev and Krachunov (2004) and by Kaishev and Dimitrova (2005). As a joint optimality criterion they introduce the probability that the direct insurer and the reinsurer jointly survive up to a finite time horizon. In this model, individual claims are assumed to have any joint severity distribution and Poisson arrivals. Ignatov, Kaishev and Krachunov (2004) have considered a simple excess of loss (XL) contract with a retention level, taking integer values, and discrete claim amounts. This model has been further extended by Kaishev and Dimitrova (2005) who consider XL reinsurance with a retention level and a limiting level and claim amounts with any continuous joint distribution. Under this more general model, explicit expressions for the probability of joint survival up to a finite time horizon have been derived.

In this paper we consider this more general model further and derive expressions for the expected profit at time x of the insurer as well as of the reinsurer, given their joint survival up to x. We prove that, under the considered XL contract, there always exists a solution to the optimality problem of splitting the total expected profit, given survival, in a proportion q, if the total premium income has been split between the insurer and the reinsurer in the same proportion. A Markowitz efficient frontier type approach to setting optimally the limiting level and the retention level is proposed and developed. Following

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this approach we view the probability of joint survival as a risk measure and the expected profit given joint survival as a performance measure. This methodology is illustrated numerically on several examples of appropriate claim amount distributions including some heavy-tailed distributions, used in extreme value modelling, both for the case of independent and dependent claim severities. We show that the derived expressions have fast convergence and coveniently allow the use of copula functions in modelling the dependency between claim severities.

Keywords: excess of loss reinsurance, probability of non-ruin, Appell polynomials, joint survival of cedent and reinsurer, expected profit, efficient frontier, copula functions

References

Aase, K. (2002). Perspectives of Risk Sharing. *Scand. Actuarial J. 2, 73-128.*

Andersen, K.M. (2000). Optimal choice of reinsurance-parameters by minimizing the ruin probability. Thesis for the Degree Cand. Act., University of Copenhagen, Laboratory of Actuarial Mathematics.

Centeno, M. L. (1991). An insight into the excess of loss retention limit. *Scand. Actuarial J., 97-102.*

Centeno, M. L. (1997). Excess of loss reinsurance and the probability of ruin in finite horizon. *ASTIN Bulletin,* 27, 1, 59-70.

Dickson, D.C.M. and Waters, H.R. (1996). Reinsurance and ruin. *Insurance: Mathematics and Economics,* 19, 1, 61-80.

Dickson, D.C.M. and Waters, H.R. (1997). Relative reinsurance retention levels. *ASTIN Bulletin,* 27, 2, 207-227.

Hipp, C. and Vogt, M. (2001). Optimal dynamic XL reinsurance. Preprint No 1/01, University of Karlsruhe.

Ignatov, Z. G. and Kaishev, V. K. (2004). A finite time ruin probability formula for continuous claim severities. *Journal of Applied Probability,* 41, 570-578.

Ignatov, Z. G., Kaishev, V. K. and Krachunov, R. S. (2004). Optimal retention levels, given the joint survival of cedent and reinsurer. *Scand. Actuarial J.*, 6, 401-430.

Kaishev, V. K. and Dimitrova, D. S. (2005). Excess of Loss Reinsurance Under Joint Survival Optimality. *Insurance:Mathemaics and Economics (under review)*

Krvavych, Y. (2001). On existence of insurer's optimal excess of loss reinsurance strategy. Paper presented at the 5th International Congress on *Insurance:Mathematics and Economics*

Schmidli, H. (2001). Optimal proportional reinsurance policies in a dynamic setting. *Scand. Actuarial J. 1, 55-68.*

Schmidli, H. (2002). On minimizing the ruin probability by investment and reinsurance. Preprint, University of Aarhus.

Suijs, J., Borm, P., De Waegenaere, A. (1998).Stochastic cooperative games in insurance. *Insurance:Mathemaics and Economics, 22, 209-228.*

Taksar, M. and Markussen, C. (2003). Optimal Dynamic Reinsurance Policies for Large Insurance Portfolios. *Finance and Stochastics*, 7, 1, 97-121.