Let $W = (W_t)_{t \geq 0}$ be a standard Wiener process and $B^H = (B^H_t)_{t \geq 0}$ be a fractional Brownian motion with the Hurst index $H$, $H \in \left(\frac{1}{2}, 1\right)$, both defined on a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t \geq 0}, P)$.

Consider a mixed version of the Black-Merton-Scholes model, i.e., a $(B,S)$-market with a bond $B$ and a stock $S$, where

$$B_t = e^{rt}, \quad S_t = e^{aW_t+bb^H_tCt}, \quad r, a, b, c \in \mathbb{R}.$$  

For a given strategy (or a portfolio) $\pi = (\beta_t, \gamma_t)_{t \geq 0}$ the capital $X = (X_t)_{t \geq 0}$ corresponding to this portfolio equals

$$X_t = B_t \cdot \beta_t + S_t \cdot \gamma_t.$$  

We make the following assumptions about the strategy $\pi$:

1) $\pi$ is self-financing, i.e.,

$$X_t = X_0 + \int_0^t \beta_s dB_s + \int_0^t \gamma_s dS_s;$$  

2) $\pi$ is of Markov type, i.e.,

$$\beta_t = \beta(S_t, t), \quad \gamma_t = \gamma(S_t, t).$$  

We say that the strategy $\pi$ has an arbitrage opportunity if there exists $T > 0$ such that

$$X_0 = 0, \quad X_T \geq 0 \ (P-a.s.), \quad P(X_T > 0) > 0.$$  

It is well-known that in the case of standard Black-Merton-Scholes model, i.e., when $b = 0$ in (1), there exists an equivalent martingale measure. This fact guarantees the absence of arbitrage. In the case of pure fractional model, i.e., when $a = 0$ in (1), the arbitrage possibility depends on the definition of stochastic integral. It is easy to give arbitrage examples in continuous time trading with self-financing strategies, if one uses the Riemann-Stieltjes integral. It was demonstrated by Rogers [1], Shiryaev [2], Dasgupta [3]. If one uses the Wick-Itô-Skorohod integral she obtains an arbitrage-free model. Pioneer papers on this topics belong to Hu, Øksendal [4] and Elliott, van der Hoek [5], then the situation was carefully studied by Bender [6] where some clarifications were introduced. The results have been summarised by Sottinen and Valkeila in [7]. In the mixed model (1) with $a \neq 0$ and $b \neq 0$ some results were obtained in the papers of Kuznetsov [8], Cheridito [9] and Mishura, Valkeila [10]. Kuznetsov proved the absence of arbitrage under condition of independence of processes $W$ and $B^H$. Cheridito established that for $H \in \left(\frac{3}{4}, 1\right)$ the
mixed model with independent $W$ and $B^H$ is equivalent to the one with Brownian motion and hence it is arbitrage-free. Conversely, the work of Mishura and Valkeila guarantees the absence of arbitrage when the trajectories of process $B^H$ are in some sense the functionals of trajectories of process $W$.

The main result of our paper is that the mixed market is arbitrage-free for any $W$ and $B^H$ if we restrict ourselves with self-financing Markov-type smooth strategies.

We introduce for $\alpha > 0$ the process $B^H, \alpha$ which is continuously differentiable and for which convergence $E \left| B^H_t - B^H_t, \alpha \right|^{\alpha} \to 0$ holds as $\alpha \to 0+$ for $2 \leq m < \frac{1}{1-H}$. Also we prove in the next section that if process $f$ belongs to the class $C^{2(1-H) + \varepsilon}$ with some $\varepsilon > 0$, then for $H \in \left(\frac{3}{4}, 1\right)$ convergence in probability $\int_0^T f(u) dB^H, \alpha_u \to \int_0^T f(u) dB^H_u$ holds as $\alpha \to 0+$, where the last integral is the Riemann-Stieltjes one. Using this auxiliary result we represent the process of capital in a mixed Black–Merton–Scholes model (1) as a limit of some semimartingales.

References