Ratio monotonicity for tail probabilities in the renewal risk model

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Abstract

A renewal model in risk theory is considered, where $\overline{H}(u,y)$ is the tail of the distribution of the deficit at ruin and $\overline{F}(u)$ is the tail of the ladder height distribution. Conditions are derived under which the ratio $\overline{H}(u,y)/\overline{F}(u+y)$ is nondecreasing in u for any $y \geq 0$. In particular, it is proven that if the ladder height distribution is stable and DFR or phase-type, then the above ratio is nondecreasing in u. As a byproduct of this monotonicity, an upper bound and an asymptotic result for $\overline{H}(u,y)$ are derived. Examples are given to illustrate the monotonicity results.

Keywords: ladder height; deficit at ruin; ruin probability; renewal equation; DFR; stable distribution; phase-type distribution; nonnegative matrix; M-matrix.

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