Enterprise Risk Management Through Strategic Allocation of Capital

Joint Work

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Overview

- Introduction to ERM and Literature Review
- Baseline One-Period Model
- The Choice of Risk Appetite
- A Numerical Illustration
- The Two-Period Model
- Discussions and Conclusion
What is ERM?

The concept of Enterprise Risk Management (ERM)

- Managing risks holistically rather than separately
- Unique features of ERM involves
  - risk appetite
  - inter-relations between risks
  - risk prioritization
  - alignment of strategic goals and risk considerations
- The users of ERM: corporations, universities, and government
What is Driving ERM?

Compliance

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Best Practice Standard
- Standard & Poor’s (2005, 2006)
- A. M. Best (2007)
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Value Creation
The ultimate goal of ERM is to create value for stakeholders
“Much ambiguity remains as to what ERM really is and how it should be implemented” (Towers and Perrin, 2006)
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- The concept
Research Motivation

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- Risk identification, risk assessment, and risk communication
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- A quantified, implementable framework
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A desired approach

- A framework for operational decisions
- Capture important characteristics
- Flexible and adaptive
Propose an ERM Framework

**Research Question**

How to formulate a quantitative ERM framework which is amenable for implementation in a general firm?
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Features of the proposed framework

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- The dynamic framework allows the firm to account for the changing business environment
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- Facilitates the integration of and interactions between strategic goals, operational decisions, and risk considerations
- Explicitly captures “risk appetite”, “inter-relation”, and “risk prioritization” in the decision making process
- The dynamic framework allows the firm to account for the changing business environment
- Provides a conceptual framework capable of facilitating more general ERM modeling
## ERM Literature

### Components of ERM

- Determination of ERM: Liebenberg and Hoyt (2003)
- (Quantile-based) risk measure: Dowd and Blake (2006)
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Chance-constrained programming approach (Charnes and Cooper, 1959, 1962, 1963)
To achieve business target in light of a multitude of risk considerations

(1) project risk ≤ project risk appetite
(2) financial risk ≤ financial risk appetite
(3) operational risk ≤ operational risk appetite
(4) hazard risk ≤ hazard risk appetite
(5) overall risk ≤ overall risk appetite
(6) other considerations (e.g., budget constraint)

- by making appropriate operational decisions
The General Idea

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- Risk considerations are traded off in stochastic constraints
  - Several major types of risks are considered
  - Use Value-at-Risk (VaR) (quantile-based) risk measure
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  - Use Value-at-Risk (VaR) (quantile-based) risk measure
- Explicitly incorporate risk appetite, inter-relations between risks, and risk prioritization
The Baseline Setting

- Single period risk/return optimization for a general firm, with total capital $C$ to allocate
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- Two sets of investment opportunities
  - $i = 1, \ldots, K$ real projects (P) (e.g., $K$ manufacturing product lines)
  - $j = 1, \ldots, N$ financial assets (A)
The Baseline Setting

- Single period risk/return optimization for a general firm, with total capital $C$ to allocate
- Two sets of investment opportunities
  - $i = 1, \ldots, K$ real projects (P) (e.g., $K$ manufacturing product lines)
  - $j = 1, \ldots, N$ financial assets (A)
- Model assumptions
  - Random returns
  - Direct loss from hazard risk is proportional to total capital
  - Indirect loss from hazard risk is a percentage of direct loss
  - Hazard risk is mitigated by insurance
Objective Function

$$\max \left[ E(\text{returns from projects}) + E(\text{returns from assets}) - \text{insurance cost} \right]$$
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$$\max_{w_i^{(P)}, w_j^{(A)}, u} \sum_{i=1}^{K} w_i^{(P)} [1 + E(r_i^{(P)})] + \sum_{j=1}^{N} w_j^{(A)} [1 + E(r_j^{(A)})] - ud(1 + \theta)\mu$$
**Objective Function**

\[
\text{max } \left[ \mathbb{E}(\text{returns from projects}) + \mathbb{E}(\text{returns from assets}) - \text{insurance cost} \right]
\]

\[
\text{max } w(P) \sum_{i=1}^{K} w_i(P) \left[ 1 + \mathbb{E}(r_i(P)) \right] + \sum_{j=1}^{N} w_j(A) \left[ 1 + \mathbb{E}(r_j(A)) \right] - ud(1+\theta)\mu
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- \(w_i(P)\): proportions of capital invested in projects
- \(r_i(P)\): random returns from projects
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- \(w_j^{(A)}\): proportions of capital invested in assets
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- \(\mu\): expected unit direct hazard loss; \(\theta\): indirect hazard loss
- \(d\): insurance loading; \(u\): proportion of total hazard risk insured
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Chance Constraints

- Account for four major types of firm risks: project risk, financial risk, operational risk, hazard risk
- Serve as the tool to incorporate the firm’s risk appetite and risk prioritization decisions
- Overall liquidity/solvency likelihood constraint specifies the likelihood of being unable to meet obligations and intertwines all the decision variables together
Project Risk Constraint

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\[ P[\text{project returns } \geq \text{ project investment} \times \text{hurdle rate}] \geq (1-\text{appetite}) \]

\[ P\left[ \sum_{i=1}^{K} w_{i}^{(P)}(1 + r_{i}^{(P)}) \geq \left( \sum_{i=1}^{K} w_{i}^{(P)}(1 + r_{0}^{(P)}) \right) \right] \geq 1 - \alpha_1 \]
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- Select projects so that a pre-determined minimum project return can be secured with certain confidence.
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- Select projects so that a pre-determined minimum project return can be secured with certain confidence
- Selection is governed by project risk appetite \( \alpha_1 \)
Financial Risk Constraint

- Different nature of project risk and financial risk
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- Natural hedges
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- Financial risk:

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P[\text{asset returns } \geq \text{ financial investment} \times \text{hurdle rate}] \geq (1 - \text{appetite})
\]

\[
P\left[ \sum_{j=1}^{N} w_j^{(A)} (1 + r_j^{(A)}) \geq \left( \sum_{j=1}^{N} w_j^{(A)} \right) (1 + r_0^{(A)}) \right] \geq 1 - \alpha_2
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Financial Risk Constraint

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- Natural hedges
- Financial risk:

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Could incorporate hedging policies in the decision framework (e.g., Caldentey and Haugh 2006)
Operational (OP) Risk Constraint

- “The most important risk category” (Towers and Perrin 2006)
Operational (OP) Risk Constraint

- “The most important risk category” (Towers and Perrin 2006)
- Standardized approach in Basel Capital Accord II

\[ OP \text{ risk} = \sum_i (\text{factor}_i \times \text{general indicator in business unit } i) \]
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P[\gamma_1 \sum_{i=1}^{K} w_i^{(P)} (1 + r_i^{(P)}) + \gamma_2 \sum_{j=1}^{N} w_j^{(A)} (1 + r_j^{(A)}) \leq l_{op}] \geq 1 - \alpha_3
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- General indicators for each business unit
- Two different factors for each business unit
- Interactions between project risk and financial risk
Hazard Risk Constraint

- Mitigate hazard risk by purchasing (costly) insurance
- One constraint to govern the financial consequences from uninsured proportion of the hazard risk
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- One constraint to govern the financial consequences from uninsured proportion of the hazard risk
- Hazard risk:

\[ P[\text{uninsured proportion} \times \text{hazard risk} \leq \text{hazard risk limit}] \geq (1-\text{appetite}) \]

\[ P[(1 - u)l' \leq m] \geq 1 - \alpha_4 \]
Hazard Risk Constraint

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- One constraint to govern the financial consequences from uninsured proportion of the hazard risk
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\[ P[(1 - u)l' \leq m] \geq 1 - \alpha_4 \]

- \( l' = l \times (1 + \theta) \): the unit total hazard loss

\[ m \] is the hazard risk limit

\[ u \] is the proportion of total hazard risk insured

\[ \alpha \] is the static risk appetite

\[ P \] is the probability
Hazard Risk Constraint

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Overall Risk Constraint

- An integration of all risk categories
- The firm needs to be able to repay the required obligations (when contingent borrowing is not possible) to avoid default
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\[ P[(\text{project: returns - op risk}) + (\text{financial: returns - op risk}) - \text{uninsured hazard risk} - \text{insurance} \leq \text{obligation}] \leq \text{(appetite)} \]
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- (1 - u)l' - (1 + d)u(1 + \theta)\mu \leq c] \leq \bar{\alpha} \]
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Deterministic Constraints

- **Budget constraint:**
  \[
  \sum_{i=1}^{K} w_i(P) + \sum_{j=1}^{N} w_j(A) + u(1 + d)(1 + \theta)\mu \leq 1
  \]

- **Strategic constraint:**
  \[
  \sum_{i=1}^{K} w_i(P) \geq \gamma_3
  \]

- **Range constraints:**
  \[
  w_i(P) \geq 0, w_j(A) \geq 0, 0 \leq \nu \leq 1
  \]
The ERM Framework

\[
\max_{w_i^{(P)}, w_j^{(A)}, u} \ E[\text{project returns} + \text{financial returns} - \text{insurance}]
\]

s.t.
\[P[\text{project returns} \geq \text{hurdle rate}] \geq (1-\alpha_1)\]
\[P[\text{financial returns} \geq \text{hurdle rate}] \geq (1-\alpha_2)\]
\[P[\text{op risk} \leq \text{risk limit}] \leq \alpha_3\]
\[P[\text{uninsured hazard risk} \leq \text{risk limit}] \leq \alpha_4\]
\[P[\text{total returns} - \text{all risk} - \text{insurance} \leq \text{obligation}] \leq \bar{\alpha}\]

budget constraint, strategic constraint, range constraints
The ERM Framework

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\max_{w_i^{(P)}, w_j^{(A)}, u} \quad E[\text{project returns + financial returns - insurance}]
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\end{align*}
\]
budget constraint, strategic constraint, range constraints
Computation of the Constraint Set

To convert from the VaR type risk constraint to a deterministic constraint for use in mathematical programming, we use the fact that if

\[ P[X \geq t] \geq 1 - \alpha \]

then

\[ t \leq F^{-1}(\alpha) \]
Computation of the Constraint Set

- To convert from the VaR type risk constraint to a deterministic constraint for use in mathematical programming, we use the fact that if

\[ P[X \geq t] \geq 1 - \alpha \]

then

\[ t \leq F^{-1}(\alpha) \]

- So for example the financial risk constraint

\[ P\left[ \sum_{j=1}^{N} w_j^{(A)}(1 + r_j^{(A)}) \geq \left( \sum_{j=1}^{N} w_j^{(A)} \right)(1 + r_0^{(A)}) \right] \geq 1 - \alpha_2 \]

becomes

\[ (1+r_0^{(A)})(W^{(A)})^T \iota - (W^{(A)})^T (\iota + E(r)^{(A)}) \leq \Phi^{-1}(\alpha_2) \sqrt{(W^{(A)})^T \Sigma^{(A)} W^{(A)}} \]
The firm should articulate how risk appetite falls in line with strategic goals and risk culture (S&P 2005)
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Determine risk appetite in light of the credit rating target (Nocco and Stulz 2006)

- Strategic goals govern the risk appetite
- Credit rating target proxies for the firm’s strategic goals
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Determine risk appetite in light of the credit rating target (Nocco and Stulz 2006)

- Strategic goals govern the risk appetite
- Credit rating target proxies for the firm’s strategic goals
  - determines the ability to raise capital and the cost of capital (West 1973)
  - influences corporate policies and actual business strategies (Sufi 2007)
Choice of Risk Appetite

- Overall risk appetite parameter ($\bar{\alpha}$)
  - target default probability implied by the credit rating target
  - used in the overall risk constraint
Choice of Risk Appetite

- Overall risk appetite parameter ($\bar{\alpha}$)
  - target default probability implied by the credit rating target
  - used in the overall risk constraint
- Individual risk appetite parameters ($\alpha'_i$s)
  - different appetites toward different risks
  - reference point: financial distress probability
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  - target default probability implied by the credit rating target used in the overall risk constraint
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- Moody’s Rate Transition Matrix

<table>
<thead>
<tr>
<th>Rating From:</th>
<th>Aaa</th>
<th>Aa</th>
<th>A</th>
<th>Baa</th>
<th>Ba</th>
<th>B</th>
<th>Caa-C</th>
<th>Default</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aaa</td>
<td>91.75%</td>
<td>7.26%</td>
<td>0.79%</td>
<td>0.17%</td>
<td>0.02%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Aa</td>
<td>1.32%</td>
<td>90.71%</td>
<td>6.92%</td>
<td>0.75%</td>
<td>0.19%</td>
<td>0.04%</td>
<td>0.01%</td>
<td>0.06%</td>
</tr>
<tr>
<td>A</td>
<td>0.08%</td>
<td>3.02%</td>
<td>90.24%</td>
<td>5.67%</td>
<td>0.76%</td>
<td>0.12%</td>
<td>0.03%</td>
<td>0.08%</td>
</tr>
<tr>
<td>Baa</td>
<td>0.05%</td>
<td>0.33%</td>
<td>5.05%</td>
<td>87.50%</td>
<td>5.72%</td>
<td>0.86%</td>
<td>0.18%</td>
<td>0.31%</td>
</tr>
<tr>
<td>Ba</td>
<td>0.01%</td>
<td>0.09%</td>
<td>0.59%</td>
<td>6.70%</td>
<td>82.58%</td>
<td>7.83%</td>
<td>0.72%</td>
<td>1.48%</td>
</tr>
<tr>
<td>B</td>
<td>0.00%</td>
<td>0.07%</td>
<td>0.20%</td>
<td>0.80%</td>
<td>7.29%</td>
<td>80.62%</td>
<td>6.23%</td>
<td>4.78%</td>
</tr>
<tr>
<td>Caa-C</td>
<td>0.00%</td>
<td>0.03%</td>
<td>0.06%</td>
<td>0.23%</td>
<td>1.07%</td>
<td>7.69%</td>
<td>75.24%</td>
<td>15.69%</td>
</tr>
</tbody>
</table>

Average one-year rating transition matrix, 1920-2005, conditional upon no rating withdrawal.

Choice of Risk Appetite

- **Overall risk appetite parameter** ($\bar{\alpha}$)
  - target default probability implied by the credit rating target
  - used in the overall risk constraint

- **Individual risk appetite parameters** ($\alpha'_i$)
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Choice of Risk Appetite

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Risk Prioritization

Individual risk appetite parameters facilitate the quantification of risk prioritization decisions:
Choice of Risk Appetite

- Overall risk appetite parameter ($\bar{\alpha}$)
  - target default probability implied by the credit rating target
  - used in the overall risk constraint
- Individual risk appetite parameters ($\alpha'_{i}s$)
  - different appetites toward different risks
  - reference point: financial distress probability

Risk Prioritization

Individual risk appetite parameters facilitate the quantification of risk prioritization decisions: smaller value indicates higher priority of the risk
Numerical Example: Baseline Setting I

Purposes

- Demonstrate how to implement the framework
- Demonstrate how to determine risk appetite parameters
Numerical Example: Baseline Setting 1

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- Demonstrate how to implement the framework
- Demonstrate how to determine risk appetite parameters

Distributional assumptions

- Returns $r_i^{(P)}$ and $r_j^{(A)}$: $N \left[ (E(r^{(P)}): E(r^{(A)}))^T, \sum^{(PA)} \right]$
Numerical Example: Baseline Setting I

Purposes
- Demonstrate how to implement the framework
- Demonstrate how to determine risk appetite parameters

Distributional assumptions
- Returns \( r_i^{(P)} \) and \( r_j^{(A)} \): \( N \left[ (E(r^{(P)}): E(r^{(A)}))^T, \sum^{(PA)} \right] \)
- Unit total losses from hazard risk \( l' \sim N((1 + \theta)\mu, (1 + \theta)^2\sigma^2), 0 < \mu < 1, \sigma > 0 \)
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- Unit total losses from hazard risk
  $l' \sim N((1 + \theta)\mu, (1 + \theta)^2\sigma^2)$, $0 < \mu < 1, \sigma > 0$
- Hazard risk is independent of the other risk categories
### Description of investment opportunities

#### Expected Return and Variance

<table>
<thead>
<tr>
<th></th>
<th>R&amp;D Project</th>
<th>Manufacturing Project</th>
<th>Index Fund</th>
<th>3 Month T-Bill</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected return</td>
<td>0.3</td>
<td>0.1</td>
<td>0.12</td>
<td>0.038</td>
</tr>
<tr>
<td>Variance</td>
<td>0.3</td>
<td>0.003</td>
<td>0.03</td>
<td>0.00025</td>
</tr>
</tbody>
</table>

#### Correlation of the investments

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<tr>
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<tbody>
<tr>
<td>R&amp;D Project</td>
<td>1</td>
<td>0.1</td>
<td>0.05</td>
<td>-0.05</td>
</tr>
<tr>
<td>Manufacturing Project</td>
<td>0.1</td>
<td>1</td>
<td>0.05</td>
<td>-0.05</td>
</tr>
<tr>
<td>Index Fund</td>
<td>0.1</td>
<td>1</td>
<td>1</td>
<td>-0.1</td>
</tr>
<tr>
<td>3 Month T-Bill</td>
<td>0.1</td>
<td>1</td>
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Assume the firm has a target credit rating of A, which leads to 0.08% default probability and around 6% financial distress probability.
Parameter Values

- Assume the firm has a target credit rating of A, which leads to 0.08% default probability and around 6% financial distress probability
- Moody's Transition Matrix

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Parameter Values

<table>
<thead>
<tr>
<th></th>
<th>Risk Appetite</th>
<th>Risk Limits</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\alpha_1$</td>
<td>$r_0^{(P)}$</td>
</tr>
<tr>
<td></td>
<td>$\alpha_2$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\alpha_3$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\alpha_4$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\bar{\alpha}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.05</td>
<td>0</td>
</tr>
<tr>
<td>Hazard Risk</td>
<td>$\mu$</td>
<td>$\gamma_1$</td>
</tr>
<tr>
<td></td>
<td>$\sigma$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\theta$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$d$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>0.01</td>
<td>0.2</td>
</tr>
</tbody>
</table>
### Optimization Results

#### Optimization Results under the Baseline Setting

<table>
<thead>
<tr>
<th>Decision Variable</th>
<th>Optimal Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Investment in the R&amp;D project</td>
<td>0.0558</td>
</tr>
<tr>
<td>Investment in the manufacturing project</td>
<td>0.6228</td>
</tr>
<tr>
<td>Investment in the index fund</td>
<td>0.0542</td>
</tr>
<tr>
<td>Investment in the Treasury bill</td>
<td>0.2546</td>
</tr>
<tr>
<td>The proportion of hazard risk insured</td>
<td>0.9479</td>
</tr>
<tr>
<td>Optimal Return</td>
<td>1.0806</td>
</tr>
</tbody>
</table>
## Risk Prioritization

### No Prioritization vs. With Prioritization

<table>
<thead>
<tr>
<th>Decision Variable</th>
<th>No Prioritization</th>
<th>With Prioritization</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All $\alpha$’s = 0.05</td>
<td>$\alpha_1 = 0.1$, $\alpha_4 = 0.01$</td>
</tr>
<tr>
<td>Investment in the R&amp;D project</td>
<td>0.0558</td>
<td>0.0991</td>
</tr>
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</tr>
<tr>
<td>Optimal Return</td>
<td>1.0806</td>
<td>1.0862</td>
</tr>
</tbody>
</table>
## Interactions Between Risks

### High Op Risk Factors vs. Low Op Risk Factors

<table>
<thead>
<tr>
<th>Decision Variable</th>
<th>High Op Factors ($\gamma_1=0.2$, $\gamma_2=0.1$)</th>
<th>Low Op Factors ($\gamma_1=0.05$, $\gamma_2=0.05$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>R&amp;D project</td>
<td>0.0558</td>
<td>0.0813</td>
</tr>
<tr>
<td>Manufacturing project</td>
<td>0.6228</td>
<td>0.9062</td>
</tr>
<tr>
<td>Index fund</td>
<td>0.0542</td>
<td>0</td>
</tr>
<tr>
<td>Treasury bill</td>
<td>0.2546</td>
<td>0</td>
</tr>
<tr>
<td>Hazard risk insured</td>
<td>0.9479</td>
<td>0.9479</td>
</tr>
<tr>
<td>Optimal Return</td>
<td>1.0806</td>
<td>1.1004</td>
</tr>
</tbody>
</table>
Two-Period Setting

- Risk/return optimization in a two-period planning horizon
- Three broad types of investment opportunities
  - Short-term (one-period) real projects: invest at the beginning of each period, return at end of each period
  - Long-term (two-period) real projects: invest at the beginning of period 1, return at end of period 2
  - Financial assets
Strategic Time Line

\[ C^{(0)} \]

Time 0

\[ r_j^{(1)} = \mathbb{E}[R_j^{(1)}|\mathcal{F}_0], \quad j = P_1 \text{ or } A \]

Time 1

\[ E[C^{(1)}] \]

Time 2

\[ r_{p2} = \mathbb{E}[R_{p2}|\mathcal{F}_0] \]

\[ r_j^{(2)} = \mathbb{E}[R_j^{(2)}|\mathcal{F}_0], \quad j = P_1 \text{ or } A \]
Strategic Time Line

Time 0
- $C^{(0)}$
- $w_{p_1}^{(1)}$, $w_{p_2}$, $w_{A_k}^{(1)}$, $u^{(1)}$

Time 1
- $r_j^{(1)} = \mathbb{E}[R_j^{(1)} | \mathcal{F}_0]$, $j = P_1$ or $A$
- $E[C^{(1)}]$
- $w_{p_1}^{(2)}$, $w_{p_2}$, $w_{A_k}^{(2)}$, $u^{(2)}$

Time 2
- $r_p^{(2)} = \mathbb{E}[R_p^{(2)} | \mathcal{F}_0]$
- $r_j^{(2)} = \mathbb{E}[R_j^{(2)} | \mathcal{F}_0]$, $j = P_1$ or $A$
Strategic Time Line

\[ \hat{r}_{Pj1}, \hat{r}_{Ak1} \]

\[ \hat{r}_{Pj1}, \hat{r}_{Ak1} \]

\[ \hat{C}^{(1)} \]

\[ s_{P2} = \mathbb{E}[R_{P2} | \mathcal{F}_i] \]

\[ s^{(2)}_j = \mathbb{E}[R^{(2)}_j | \mathcal{F}_i], \quad j = P_1 \text{ or } A \]
Strategic Time Line

\[ s_{p_2} = E[R_{p_2} | \mathcal{F}_t] \]

\[ s_j^{(2)} = E[R_j^{(2)} | \mathcal{F}_t], \quad j = P_1 \text{ or } A \]
Strategic Time Line

Time 0

$w^{(1)}_{R_{j_1}}$, $w^{(1)}_{A_{k_1}}$, $w^*_P$, $u^*_1$

Time 1

$\hat{r}^{(1)}_{R_{j_1}}$, $\hat{r}^{(1)}_{A_{k_1}}$

Time 2

$\hat{s}^{(2)}_{R_{j_2}}$, $\hat{s}^{(2)}_{A_{k_2}}$, $\hat{s}_{P_{j_3}}$

$C^{(2)}$
The Dynamic Framework

- Stage 1 (based on information set $I_0$)

  \[ \text{max } E[\text{end-of-horizon total return}] \]

  s.t. Each risk constraint for period 1

  Each risk constraint for period 2

  Budget constraint, Strategic constraint, Range constraint

- Stage 2 (based on information set $I_1$)

  \[ \text{max } E[\text{end-of-horizon total return}] \]

  s.t. Each risk constraint for period 2

  Budget constraint, Strategic constraint, Range constraint
Discussions

- Distributional assumptions
  - Elliptically symmetric distribution (convex programming)
  - The copula method (search methods)

- Risk management strategies

- Decision of risk appetite

- Other model extensions
  - Multi-stage model
  - Other stochastic optimization models