Population heterogeneity in Defined Contribution Pension Schemes

Valeria D’AMATO\textsuperscript{a}, Gabriella PISCOPO\textsuperscript{b}, Maria RUSSOLILLO\textsuperscript{a}

\textsuperscript{a} Department of Economics and Statistics, University of Salerno, e-mail: vdamato@unisa.it, mrussolillo@unisa.it

\textsuperscript{b} Department of Mathematics and Statistics, University of Napoli Federico II, e-mail: gabriella.piscopo@unina.it
• The aim
• The Defined Contribution Pension Scheme
• The Mortality Heterogeneity
• The Poisson LC model
• Numerical Application
• Concluding Remarks
• Main References
Index

• The aim
• The Defined Contribution Pension Scheme
• The Mortality Heterogeneity
• The Poisson LC model
• Numerical Application
• Concluding Remarks
• Main References
The aim

To catch mortality heterogeneity and to value its impact on Pension system

- Population Ageing
- Sustainability of Pension System
Index

- The aim
- The Defined Contribution Pension Scheme
- The Mortality Heterogeneity
- The Poisson LC model
- Numerical Application
- Concluding Remarks
- Main References
The Defined Contribution Pension Scheme

• L 335/1995: Notional Defined Contribution (NDC)

  ➢ Notional accumulated contributions on individual accounts are converted into an annuity at retirement

  ➢ Total amount of contribution paid during the working life
  ➢ Life expectancy at retirement
  ➢ Survivors’ benefits
The Defined Contribution Pension Scheme

\[
P(x) = \left[ c_a + \sum_{i=1}^{a-1} c_i \prod_{j=1}^{a-1} (1 + \bar{g}_j) \right] \delta_x
\]

where

- \( x \) is the retirement age
- \( c_i \) is the contribution paid at seniority \( i \),
- \( a \) is the seniority at retirement ,
- \( \bar{g}_j \) is the geometric mean of GDP growth rate according the 5 years preceding \( j \)
- \( \delta_x \) is the transformation coefficient
The transformation Coefficient

\[ \delta_x = \left( \frac{\sum_{s=m,f} \text{dir}_{x,s} + \text{ind}_{x,s}}{2} - \gamma \right)^{-1}; \quad x \in [57,65] \]

\[ \text{dir}_{x,s} = \sum_{t=0}^{(\Omega-x)} \frac{l_{x+t,s}}{l_{x,s}} (1 + g_f)^{-t} \]

\[ \text{ind}_{x,s} = \theta \sum_{t=0}^{(\Omega-x)} \frac{l_{x+t,s}}{l_{x,s}} \left(1 - \frac{l_{x+t+1,s}}{l_{x+t,s}}\right) (1 + g_f)^{-(t+1)} a_{x+t+1}^W \]

where
\[ \gamma \] is a factor fixed by law to take into account different frequencies in pension payment
\[ g_f \] is the long-run expected GDP growth rate
\[ \theta \] is the part of pension revertible to widow(er)
The Law evolution

Law 335/95
- Transformation coefficients
- Updated every 10 years

Law 247/07
- 1st of January 2010
- New coefficients applied

Next revision of criteria will take place in 2013
• differences between genders are averaged out and transformation coefficients are the same for males and females with the same age

• redistribution between genders in a “solidaristic” way
Index

• The aim
• The Defined Contribution Pension Scheme
• The Mortality Heterogeneity
• The Poisson LC model
• Numerical Application
• Concluding Remarks
• Main References
The mortality heterogeneity

• Individuals are different with respect to mortality due to different race, geographical area and so on

• Individuals at the same age may differ in their endowment for longevity and this differences are important to population-based mortality studies

• The differences in mortality rates can compromise accuracy in the mortality projection

• Actuarial valuation can be warped if heterogeneity is not considered
Some remarks on Heterogeneity

• Important for the NDC systems rules to incorporate in the pension formulae life expectancies

• If the heterogeneity within the population is not taken into account, there is a redistribution from shorter to longer living individuals

• It can be considered desirable in a “solidaristic” view of PPP, in others it cannot
Our Research

- To provide transformation coefficients for 2013

- To take into account the heterogeneity in the data depending on the geographical area

- To derive different survival probabilities for the different geographical area

- To exploit the obtained survival probabilities for generating more actual transformation coefficients
Index

• The aim
• The Defined Contribution Pension Scheme
• The Mortality Heterogeneity
• The Poisson LC model
• Numerical Application
• Concluding Remarks
• Main References
The Lee Carter Model

\[ \ln(m_{x,t}) = \alpha_x + \beta_x k_t + \varepsilon_{x,t} \]

where

- \( m_{x,t} \) is the death rate in the year \( t \) for a policyholder aged \( x \)
- \( \alpha_x \) is an age-specific parameter independent of time
- \( k_t \) is a time-varying parameter reflecting the general level of mortality
- \( \beta_x \) describes the tendency of mortality at age \( x \) to change when the general level of mortality \( (k_t) \) changes
- \( \varepsilon_{x,t} \) is the error term, assumed to be homoscedastic
The Poisson LC Model

The parameters are estimated under the assumption: $D_{x,t}$ are distributed according to the Poisson distributions

$$D_{x,t} \approx \text{Poisson}(E_{x,t} \mu_{x,t}) \quad \mu_{x,t} = \exp(\alpha_x + \beta_x k_t)$$

$E_{x,t}$ the number of person years from which $D_{x,t}$ occurred
Index

• The aim
• The Defined Contribution Pension Scheme
• The Mortality Heterogeneity
• The Poisson LC model
• Numerical Application
• Concluding Remarks
• Main References
Application to Italian mortality rates

- Consider sub-populations (North, Center, South) so that we work on more homogeneous groups

- Implement an iterative regression methodology in R for the analysis of age-period mortality data

- Produce forecasts of future mortality rates and compute the corresponding future life expectancy for North, Center, South

- Estimate transformation coefficients for 2013 by using survival probabilities for area
The LC Poisson fitted on Italian Data

**Figure 1:** The parameter estimates of LCP on Italian male mortality data

**Figure 2:** The parameter estimates of LCP on Italian female mortality data
The LC Poisson fitted on Sub-Populations

Figure 1: The parameter estimates of LCP on North male mortality data

Figure 2: The parameter estimates of LCP on South male mortality data
The LC Poisson fitted on Sub-Populations

**Figure 1**: The parameter estimates of LCP on North female mortality data

**Figure 2**: The parameter estimates of LCP on South female mortality data
The Forecasts

**Figure 1:** Life expectancy at 60 for male (North population)

**Figure 2:** Life expectancy at 60 for male (South population)
The Forecasts

Forecasts from Random walk with drift
North : female

Forecasts of Life Expectancy at age 60
North : female

Forecasts from Random walk with drift
South : female

Forecasts of Life Expectancy at age 60
South : female

**Figure 1**: Life expectancy at 60 for male

**Figure 2**: Life expectancy at 60 for female
The projected transformation coefficients

<table>
<thead>
<tr>
<th>Level</th>
<th>57</th>
<th>58</th>
<th>59</th>
<th>60</th>
<th>61</th>
<th>62</th>
<th>63</th>
<th>64</th>
<th>65</th>
</tr>
</thead>
<tbody>
<tr>
<td>nord</td>
<td>4.265%</td>
<td>4.330%</td>
<td>4.397%</td>
<td>4.465%</td>
<td>4.534%</td>
<td>4.603%</td>
<td>4.674%</td>
<td>4.745%</td>
<td>4.818%</td>
</tr>
<tr>
<td>centro</td>
<td>4.261%</td>
<td>4.326%</td>
<td>4.392%</td>
<td>4.460%</td>
<td>4.529%</td>
<td>4.599%</td>
<td>4.670%</td>
<td>4.742%</td>
<td>4.814%</td>
</tr>
<tr>
<td>sud</td>
<td>4.354%</td>
<td>4.420%</td>
<td>4.488%</td>
<td>4.557%</td>
<td>4.627%</td>
<td>4.697%</td>
<td>4.768%</td>
<td>4.840%</td>
<td>4.912%</td>
</tr>
</tbody>
</table>

We have derived estimations of coefficients for 2013, setting according to the Law:

\[ g_f = r = 0.015 \]

\[ \theta = 0.6 \]

\[ \gamma = 0.42 \]
The projected transformation coefficients
Index

- The aim
- The Defined Contribution Pension Scheme
- The Mortality Heterogeneity
- The Poisson LC model
- Numerical Application
- Concluding Remarks
- Main References
Concluding Remarks

• According to the actual Law TC are the same

• If the heterogeneity within the population is not taken into account, there is a redistribution from shorter to longer living individuals

• If we take into account the heterogeneity in the data, higher transformation coefficients should be applied

• To have higher pension for shorter period
Index

- The aim
- The Defined Contribution Pension Scheme
- The Mortality Heterogeneity
- The Poisson LC model
- Numerical Application
- Concluding Remarks
- Main References
Main References


