A Savings Plan with Targeted Contributions

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Abstract

We consider a simple savings problem where contributions are made to a fund and invested to meet a future liability. The conventional approach is to estimate future investment return and calculate a fixed contribution to be paid regularly by the saver. We propose a flexible plan where contributions are systematically adjusted and targeted. We show by means of stochastic simulations that this plan has a reduced risk of a shortfall and is relatively insensitive to errors in the planner's estimate of future returns. Sensitivity analyses in terms of parameter values, stochastic return models and investment horizons are also performed.

Keywords: Savings plan, contributions, pension plans

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Introduction

We consider a simple savings problem where an individual puts aside funds in order to meet a certain liability at a given date in the future. The individual may contribute to a medium-term investment and savings vehicle, for example, to meet school or college fees for his children in 5 years' time. Another example is where the individual wishes to purchase property. He may save for, say 5 years, and then withdraw some or all of his investment and use it as a deposit or down-payment as part of a home loan. A longer-term example is a retirement or pension fund, where the individual saves out of labour income to provide a lump sum at retirement. This may then be used to buy an annuity.

In most problems of this kind, a financial planner or adviser will assist the individual to determine how he can save and invest every month (say), depending on his household finances. Two related decisions must be made: how much the monthly contribution will be, and where the savings will be invested. In this paper, we do not consider the second decision, i.e. asset allocation, although we discuss this briefly later. We assume instead that the individual saves in a variable-rate bank account or in a low-risk mutual fund.

A very risk-averse investor could save in a term-deposit bank account, or in a series of zero-coupon bonds, yielding fixed interest rates. However, it could be that a medium-term fixed-income investment like this is unavailable or pays unattractive rates. A less risk-averse investor, such as a young worker aspiring to home-ownership, may instead wish to invest in a stock market fund to try and achieve the largest possible future down-payment as part of a home loan. If his stock market investment performs poorly after 5 years, this may mean that a smaller home loan, or a more expensive loan with a higher debt-to-equity ratio, will be available to him. On the other hand, a worker who is close to retirement age is usually advised to invest in less risky, bond-like, assets so as to secure a comfortable retirement.

In standard economic theory, this savings problem can be cast as a consumption-investment problem, with consumption (or saving) and asset allocation as decision variables, and with a utility or loss function as an objective criterion that is optimized dynamically.

In practice, however, financial planners do not employ stochastic dynamic optimization to provide retail advice to individual savers. They will suggest instead that the individual sets aside a level amount every month, or possibly a fixed percentage of income. Dynamic optimization is not used regularly by financial planners at a retail level because of the complications caused by real-world features such as taxes, transaction costs and investment charges, because of imperfect knowledge about asset return distributions, and because it is difficult to capture individuals' varying financial circumstances and requirements.

One example is in the deterministic lifestyling strategies commonly employed in target-date funds and in pension planning (Shiller, 2005; Blake, Cairns and Dowd, 2001). Advisers typically suggest a fixed monthly contribution based on a range of assumed investment rates of return (Employee Benefits Security Administration, 2006). Another example is work-related savings vehicles, such as an employer-sponsored pension plan, where a fixed proportion of salary is determined (McGill *et al.*, 2004). The new Florida public sector pension plan described by Lachance, Mitchell and Smetters (2003) requires that individuals save a uniform 9% of pay. The Actuarial Foundation and WISER (2004) suggest a rule of thumb of saving 15% of pay towards retirement.

Our approach in this paper is to try to improve upon the conventional fixed-contribution approach. We suggest a flexible targeted-contribution savings plan and show, by means of simulations, that this plan is less risky than the conventional plan. Our proposed savings plan is based on a method used in industrial process control (for example, Box and Luceño, 1995). The method has also been proposed in the econophysics literature (Gandolfi, Sabatini and Rossolini, 2007) and is applied in the pensions literature to defined benefit pension funding with deterministic economic scenarios (Owadally, 2003). The innovation in this paper is to apply this method to a savings problem with a simple stochastic investment environment.

In the next section, we describe the targeted-contribution plan. We then run stochastic simulations with serially independent normally distributed log-returns to compare the performance of this plan over a 5-year saving period with a conventional plan. We measure the risk that end-of-period wealth falls short of the target liability using both the standard deviation and 95th percentile. The sensitivity of our proposed plan to the financial planner's assumptions is verified by repeating the simulations with different parameter values. We also run simulations with constraints imposed on the flexible contributions, with non-Gaussian investment returns incorporating jumps, and with serially correlated returns. Finally, we consider an application in pension funding with a long investment horizon and with a bootstrap stochastic asset model using 59 years of investment data on equities and bonds.

Fixed- and Targeted-Contribution Plans

We set up and compare two savings plans here. The first plan that we consider is a conventional savings plan where a level contribution is paid regularly. We refer to this as a fixed-contribution plan. Under this plan, an individual decides on a target fund value F to be reached at a time horizon T (in months). A financial planner makes an estimate of the future monthly rate of return. We denote this estimate or return assumption by i_A. The individual contributes a constant amount C every month, which is calculated by amortizing F at rate i_A over T months, i.e. the stream of monthly contributions in advance accumulate at rate i_A to F.

$$F \;=\; C\,(1+i_A)i_A^{-1}((1+i_A)^T-1) \qquad (1)$$

If the fund is invested in risky securities, then the actual rate of return on the fund, denoted by i_t in month $(t-1,\, t)$, is random. Let the value of the fund at time t be F_t, with $F_0 = 0$. Then,

$$F_{t+1} \;=\; (1 + i_{t+1})\,(F_t + C) \tag{2}$$

The value F_T of the fund at time T is random and, except by chance, will differ from the desired fund target F. A terminal or final deficit will therefore emerge at time T:

$$D_T \;=\; F - F_T. \tag{3}$$

The risk for the individual saver is that there is a large deficit at time T. (A surplus is just a negative deficit here.)

The second plan that we consider is a targeted-contribution plan. This is based on the pension funding method discussed by Owadally (2003) and adapted from industrial process control (Box and Luceño, 1995). Although we do not consider the asset allocation problem in this paper, it is worth noting that Gandolfi, Sabatini and Rossolini (2007) propose a similar method for tactical asset allocation in an investment portfolio.

Under the targeted-contribution method, the individual agrees to vary his monthly contribution payment C_t for month $(t,\, t+1)$ as follows:

$$C_t \;=\; C \;+\; \lambda_1 D_t \;+\; \lambda_2 \sum_{j=0}^{\infty} D_{t-j} \tag{4}$$

where λ_1 and λ_2 are variables to be specified subject to the constraints $0 < \lambda_1 < 1$ and $0 < \lambda_2 < 1$. Here D_t represents the notional deficit at time t in the fund, relative to what the value of the fund would be if the individual follows the fixed-contribution plan and if the fund earns the anticipated return i_A every month.

$$D_t \;=\; C\,(1 + i_A) i_A^{-1} ((1 + i_A)^t - 1) \;-\; F_t \tag{5}$$

(Assume that $D_t = 0$ for $t < 0$.)

Equation (4) therefore requires that, at time t, an overpayment at a rate of λ_1 is made based on the notional deficit D_t. If actual returns on the fund are persistently lower than the anticipated return i_A, then deficits will recur systematically. Therefore, equation (4) also imposes a further overpayment, at a rate of λ_2, based on the cumulative sum of deficits up to time t. If there is no persisting deficit, i.e. positive and negative deficits cancel each other out on average, then the additional overpayment represented by the third term on the right hand side of equation (4) is on average zero.

Results of Stochastic Simulations

Stochastic simulations are performed to compare the terminal deficits under the two plans described in the preceding section. An investment horizon of $T = 60$ months and a target fund of $F = 100$ are assumed, along with values $\lambda_1 = 0.2$ and $\lambda_2 = 0.01$ in the targeted-contribution plan in equation (4).

The monthly logarithmic return $\delta_t = \ln(1 + i_t)$ on the fund is simulated as an independent and identically distributed (iid) sequence of normally distributed random numbers with mean 0.003792 and standard deviation 0.02. The mean arithmetic return is thus 0.4% per month.

A financial planner has to estimate future investment return and make an assumption i_A as to the return on the fund. Since he does not have perfect foresight, he may over- or under-estimate future average investment return. For example, if $i_A = 0.7\%$, the planner overestimates the mean return by 0.3% per month. Henceforth, we define the estimation error as the excess of i_A over the average return on the fund. When $i_A = 0.7\%$, the planner's estimation error is therefore 0.3% per month.

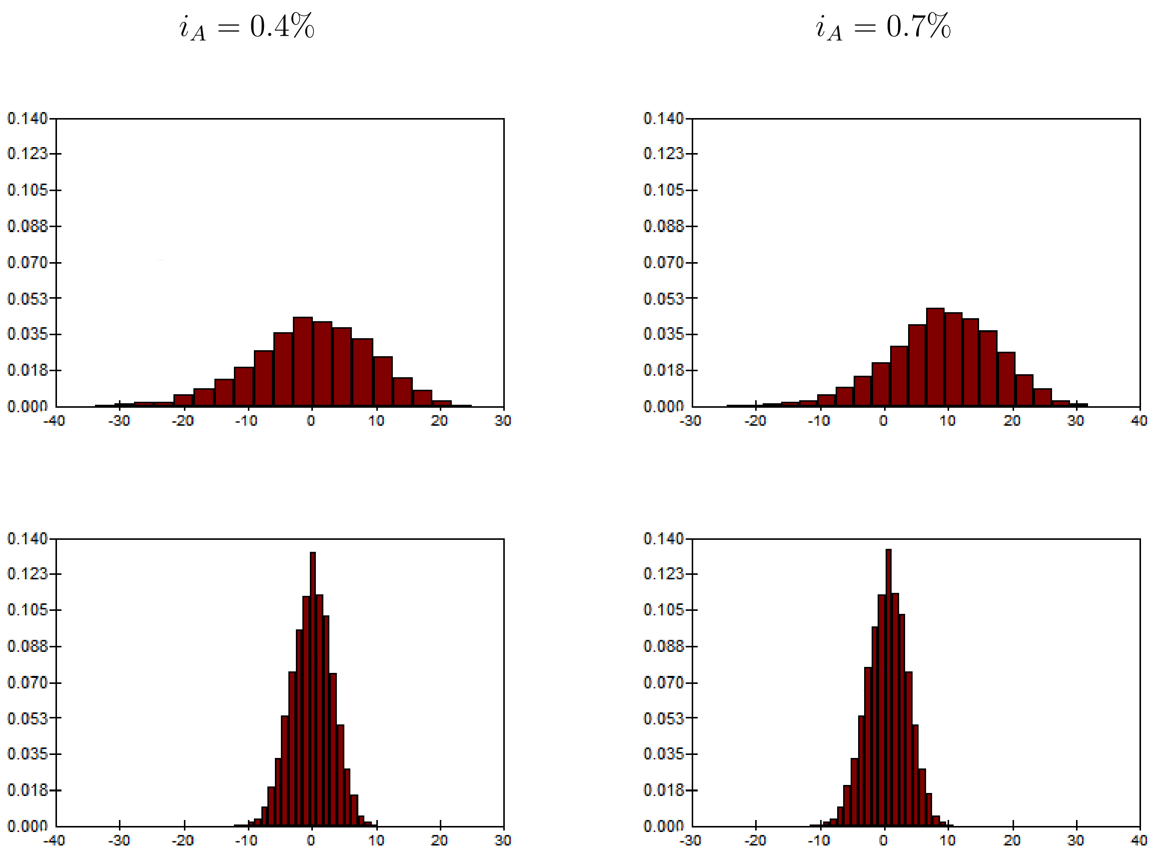

Figure 1: Histograms for the terminal deficit under fixed-contribution plan (top panels) and targeted-contribution plan (bottom panels). The target fund is 100.

Histograms of Terminal Deficit

Figure 1 shows the distributions of the terminal deficit under the two plans, for $i_A = 0.4\%$ and 0.7%. Since the accumulated fund has a right-skewed distribution, the distribution of the deficit is left-skewed. We make two observations.

OBSERVATION 1 *The fixed-contribution plan is riskier than the targeted-contribution plan (where we measure risk in terms of the fund falling short of the target at the end-date).*

The terminal deficit is more dispersed under the fixed-contribution plan (top panels in Figure 1) than under the targeted-contribution plan (bottom panels in Figure 1). The fixed-contribution plan is riskier than the targeted-contribution plan in that end-wealth is more volatile in the former case than in the latter case.

Observation 2 *Risk in the fixed-contribution plan is sensitive to the error in the financial planner's estimate of future investment return, whereas risk in the targeted-contribution plan is relatively insensitive to this. (Again, we measure risk in terms of the shortfall or deficit from the target at the end.)*

When the return assumption is 0.7%, rather than 0.4%, the adviser's return assumption overstates the mean return on the fund and we anticipate that the plans will, on average, fall short of the target. The distribution of the terminal deficit does indeed shift to the right when the return assumption changes from 0.4% to 0.7% in the fixed-contribution plan. However, it remains centered around zero in the targeted-contribution plan. If the planner's return assumption turns out to be over-optimistic, the risk of a shortfall is greater in the fixed-contribution plan than in the targeted-contribution plan.

Percentiles and Moments of Terminal Deficit

Table 1 shows how various statistics concerning the terminal deficit change as the estimation error in i_A changes.

First, note that the 95th percentile and standard deviation of the terminal deficit are smaller in the targeted-contribution plan than in the fixed-contribution plan for all values of errors in i_A in Table 1. The histograms in Figure 1 illustrate this visually for errors of 0 and 0.3%. This confirms Observation 1 above.

Secondly, note that the more i_A overestimates the average return (i.e. the greater the error in i_A), the greater the terminal deficit in the fixed-contribution plan: both the mean

Error in i_A	Mean deficit		St. deviation of deficit		95th percentile of deficit		Mean square deficit	
	FC	*TC*	*FC*	*TC*	*FC*	*TC*	*FC*	*TC*
-0.3%	-9.841	-0.497	10.385	3.223	6.219	4.697	204.701	10.636
-0.2%	-6.499	-0.336	10.068	3.214	9.073	4.843	143.600	10.442
-0.1%	-3.230	-0.171	9.760	3.205	11.863	4.998	105.691	10.299
0	-0.036	-0.003	9.458	3.195	14.591	5.155	89.453	10.210
0.1%	3.086	0.169	9.163	3.186	17.256	5.315	93.478	10.178
0.2%	6.135	0.344	8.875	3.176	19.859	5.472	116.394	10.207
0.3%	9.112	0.523	8.593	3.167	22.401	5.641	156.866	10.301

Table 1: Mean, standard deviation, mean square and 95th percentile of terminal deficits under fixed-contribution plan (FC) and targeted-contribution plan (TC). The target fund is 100. The error in i_A is the excess of i_A over the average return.

deficit and the 95th percentile of the deficit increase. The targeted-contribution plan, however, is much less sensitive to the error in i_A: the mean, standard deviation and 95th percentile of the deficit change, but not as much as in the fixed-contribution case. This confirms Observation 2 above. The mean square deficit column in Table 1 gives the second non-central moment of the deficit around zero. We note that the mean square deficit in the targeted-contribution plan is fairly constant regardless of the size of the error.

Discussion of Results and Sensitivity Analysis

Observations 1 and 2

Observation 1, i.e. that the targeted-contribution plan is less risky than the fixed-contribution plan, in terms of meeting the target at the investment horizon, is not surprising. The targeted-contribution plan is flexible and contributions can be varied to make up any shortfall gradually. In other words, the flexible contributions pick up part of the investment risk, thereby reducing the risk to end-wealth.

Observation 2, i.e. that the targeted-contribution plan is relatively insensitive to errors in the planner's estimate of investment return, is more surprising. Whereas the first observation could arise from any plan that allows flexible contributions, the second observation requires that the contributions be flexible *and* be targeted systematically, so as to counter the effect of the estimation error. The third term on the right hand side of equation (4) ensures that any notional deficit that accumulates over time is 'taxed' at a rate λ_2, thereby adjusting the contribution to restore the savings plan on a path to target.

This is illustrated in Table 2 which shows the average path of the two plans over time for a return assumption of 0.7%. The fund builds up gradually from 0 to about 90 on average in the fixed-contribution plan, leaving an average terminal shortfall of 10. By contrast, the fund builds up to nearly 100 on average in the targeted-contribution plan, leaving

time	Mean fund		Mean contribution	
(months)	*FC*	*TC*	*FC*	*TC*
0	0.0000	0.0000	1.3375	1.3375
5	6.7683	6.7833	1.3375	1.3478
10	13.6729	13.7782	1.3375	1.3677
15	20.7170	21.0242	1.3375	1.3914
20	27.9042	28.5406	1.3375	1.4166
25	35.2363	36.3379	1.3375	1.4423
30	42.7163	44.4238	1.3375	1.4685
35	50.3483	52.8063	1.3375	1.4951
40	58.1330	61.4937	1.3375	1.5220
45	66.0791	70.4943	1.3375	1.5496
50	74.1847	79.8185	1.3375	1.5778
55	82.4504	89.4747	1.3375	1.6071
60	90.8881	99.4771	1.3375	1.6369

Table 2: Mean values of fund and contribution over time under fixed-contribution plan (FC) and targeted-contribution plan (TC). The target fund is 100 and the investment return assumption overestimates actual returns by 0.3%.

almost no terminal shortfall on average. Whereas the contribution is constant under the fixed-contribution plan, the mean contribution in the targeted-contribution plan gradually increases so that the plan is on average on target after 60 months.

An estimation error in the financial planner's investment return assumption can arise in several ways. The planner's model of asset returns may be wrongly calibrated because of insufficient or inaccurate data (parameter risk). The model itself may be mis-specified (model risk). For example, the planner may fit a normal distribution to asset returns that, in fact, have a heavy-tailed distribution, thereby discounting the effect of market corrections and underestimating tail risk. An estimation error may also occur because of a large unexpected shift in the economic environment (such as after a market crash or following a revision in monetary policy targets by the central bank). Finally, and more subtly, the financial planner may exhibit behavioral biases, leading to overconfidence in an economic boom (Akerlof and Shiller, 2009; Kahneman and Riepe, 1998) and decisions influenced by framing and mental accounting issues (Thaler, 1999). Observation 2 therefore suggests that the targeted-contribution plan performs fairly robustly regardless of all these sources of estimation errors in the future investment return.

Terminal Wealth and Interim Consumption

The targeted-contribution savings plan shifts uncertainty from terminal wealth to intermediate contributions. For a short-term savings plan, or one where contribution is a small proportion of income, the variability in contributions may have a negligible impact on the saver and on the saver's consumption during the saving period. On the other hand, for a long-term savings plan, or one where the contribution is a large proportion of income, large contribution requirements over a long period could be unaffordable.

To investigate this, we repeat the simulations of the earlier section with an unbiased

	Mean deficit		St. deviation of deficit		95th percentile of deficit	
	FC	*TC*	*FC*	*TC*	*FC*	*TC*
no constraint	-0.036	-0.003	9.458	3.195	14.591	5.155
$u = 2$	-0.036	-0.015	9.458	3.249	14.591	5.185
$u = 1.75$	-0.036	0.044	9.458	3.310	14.591	5.465
$u = 1.5$	-0.036	0.270	9.458	3.530	14.591	6.189
$u = 1.25$	-0.036	1.072	9.458	4.215	14.591	8.560
$u = 1.1$	-0.036	2.451	9.458	5.153	14.591	11.806
$u = 1.05$	-0.036	3.313	9.458	5.631	14.591	13.397

Table 3: Mean, standard deviation and 95th percentile of terminal deficits under fixed-contribution plan (FC) and targeted-contribution plan (TC), for various upper constraint factor u. FC is independent of u but statistics are shown for comparison. $\lambda_1 = 0.2$, $\lambda_2 = 0.01$, $i_A = 0.4\%$.

estimate $i_A = 0.4\%$, except that we use a constrained contribution $\widetilde{C}_t$ in the targeted-contribution plan:

$$\widetilde{C}_t = \begin{cases} 0 & \text{if } C_t < 0 \\ u\,C & \text{if } C_t > u\,C \\ C_t & \text{otherwise} \end{cases} \tag{6}$$

where C_t is as in equation (4), C is the planned contribution in the fixed-contribution savings plan, and u provides an upper constraint relative to C, with $u \geq 1$. The larger u is, the less budget-constrained the saver is, i.e. the more he is able to afford contributions that are larger than planned if investment returns are unfavourable.

We anticipate that, the more budget-constrained the saver is, the more increases in contribution will be limited, and the less effective the targeted-contribution plan will be in compensating for lower than anticipated investment returns. Table 3 does indeed show that, the lower u is, the more end-wealth risk increases in the targeted-contribution plan: the volatility and 95th percentile of terminal deficits in the targeted-contribution plan increase with decreasing u.

The upper constraint u does not affect the fixed-contribution plan, of course, but statistics are shown in Table 3 for the sake of comparison. We observe from Table 3 that the targeted-contribution plan, even when constrained, performs better than the fixed-contribution plan in terms of achieving a lower end-wealth risk. It is also of practical interest to note that, even if contributions in the targeted-contribution plan are constrained to be no more than 110% of the planned contribution under the conventional fixed-contribution plan (and no less than zero), the standard deviation of the terminal deficit is reduced by almost half.

Constraining contributions as in equation (6) may be realistic to the extent that individuals are able to budget and form a view of an upper limit as to how much they can save every month. Nevertheless, the analysis in this paper is limited in that risk is measured in terms of terminal wealth only. We note, as a direction for future research, that one could consider utility over both end-wealth and the interim consumption stream. Labour income risk then becomes relevant (Campbell and Viceira, 2002). If labour income is inversely correlated with investment return, a hedging effect is induced which may reduce combined risk in terms of both consumption and end-wealth.

The intermediate consumption pattern also matters in conventional fixed-contribution plans, in fact. A prudent financial adviser will underestimate future investment return (that is, he will end up with negative values of the estimation error in Table 1) so as to minimize the 95th percentile of deficits. However, this can also result in large surpluses, in the individual making larger contribution payments than he can afford, and in a consequent

λ_1	Mean deficit		St. deviation of deficit		95th percentile of deficit	
	FC	*TC*	*FC*	*TC*	*FC*	*TC*
0	-0.0356	-0.0075	9.4579	6.7260	14.5908	10.5769
0.05	-0.0356	-0.0050	9.4579	4.9765	14.5908	7.8329
0.1	-0.0356	-0.0038	9.4579	4.0842	14.5908	6.5129
0.15	-0.0356	-0.0032	9.4579	3.5510	14.5908	5.7057
0.2	-0.0356	-0.0028	9.4579	3.1953	14.5908	5.1551
0.25	-0.0356	-0.0025	9.4579	2.9403	14.5908	4.7213

Table 4: Mean, standard deviation and 95th percentile of terminal deficits under targeted-contribution plan (TC), for various values of λ_1, and for $\lambda_2 = 0.01$. Fixed-contribution plan (FC) is independent of λ_1 but statistics are shown for comparison. $i_A = 0.4\%$.

loss in utility of consumption during the saving period. Again, we note this here as an item for further research.

Variables λ_1 and λ_2

From the discussion in the preceding section, we anticipate that increasing values of λ_1 and λ_2 in equation (4) should lead to lower risk to end-wealth in the targeted-contribution plan. We repeat the simulations of the earlier section, without constraining contributions. Tables 4 and 5 show how the risk of a deficit changes as λ_1 and λ_2 respectively change. It is noteworthy that the targeted-contribution plan has smaller and less volatile terminal deficits than the fixed-contribution plan for all values of λ_1 and λ_2 shown.

Table 4 shows indeed that the standard deviation and 95th percentile of deficit decrease as λ_1 increases. Table 5 shows the same as λ_2 increases, for low values of λ_2. However,

λ_2	Mean deficit		St. deviation of deficit		95th percentile of deficit	
	FC	*TC*	*FC*	*TC*	*FC*	*TC*
0	9.1119	1.3856	8.5931	3.2492	22.4008	6.5924
0.005	9.1119	0.8151	8.5931	3.1864	22.4008	5.9590
0.01	9.1119	0.5230	8.5931	3.1667	22.4008	5.6414
0.02	9.1119	0.2818	8.5931	3.1510	22.4008	5.3726
0.03	9.1119	0.1922	8.5931	3.1441	22.4008	5.3180
0.04	9.1119	0.1456	8.5931	3.1407	22.4008	5.1856
0.05	9.1119	0.1169	8.5931	3.1398	22.4008	5.1721
0.1	9.1119	0.0059	8.5931	3.1586	22.4008	5.0992
0.2	9.1119	0.0030	8.5931	3.2185	22.4008	5.1633
0.3	9.1119	0.0020	8.5931	3.2611	22.4008	5.2788
0.4	9.1119	0.0014	8.5931	3.3056	22.4008	5.3811
0.5	9.1119	0.0011	8.5931	3.3652	22.4008	5.5030

Table 5: Mean, standard deviation and 95th percentile of terminal deficits under targeted-contribution plan (TC), for various values of λ_2, and for $\lambda_1 = 0.1$. Fixed-contribution plan (FC) is independent of λ_2 but statistics are shown for comparison. $i_A = 0.7\%$.

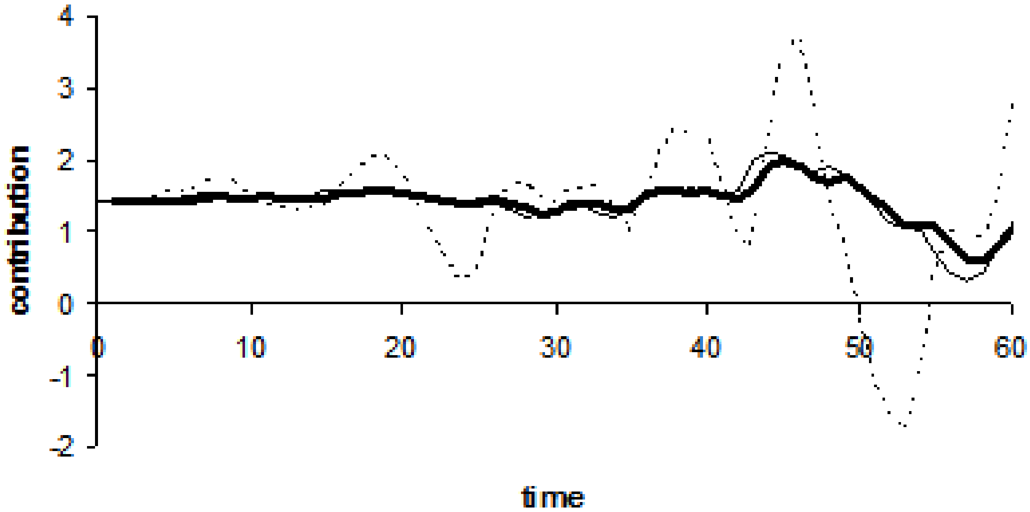

Figure 2: Sample paths of contribution in targeted-contribution plan for $\lambda_2 = 0$ (bold), $\lambda_2 = 0.01$ (continuous) and $\lambda_2 = 0.3$ (broken). The three sample paths correspond to the same sample path of investment returns on the fund, with $\lambda_1 = 0.1$.

for larger values of λ_2 (say greater than 0.1), risk appears to increase as λ_2 increases. For such large values of λ_2, the targeted-contribution plan appears to overcompensate for past deficits leading to large undesirable swings in contribution payments.

This is illustrated in Figure 2 which shows three sample paths for contribution in the targeted-contribution plan, for λ_2 =0, 0.01 and 0.3. As anticipated, there is more variability in the contributions required from the investor in the targeted-contribution plan when $\lambda_2 = 0.3$ than when $\lambda_2 = 0.01$. When $\lambda_2 = 0.3$, contributions are briefly negative so that the investor is actually required to withdraw monies from the fund.

A practical implication for financial advisers is therefore that large values of λ_1 and λ_2 in equation (4) should be avoided as they could lead to very volatile contributions and to loss of utility of consumption for the individual saver. In particular, large values of λ_2 (say, greater than 0.1) should be avoided.

Asset Return Model and Asset Allocation

In the earlier sections, we assumed that asset returns were normally distributed and serially independent. We investigate the relative performance of the fixed and targeted-contribution plans under two alternative investment return models here.

First, we consider leptokurtic asset returns by incorporating "jumps" in returns through a discrete-time version of a Lévy process. The jumps represent market corrections and crashes. We assume that the log-return $\delta_t = \ln(1 + i_t)$ on the fund in month $(t-1,\, t)$ is given by

$$\delta_t \;=\; \mu + \epsilon_t + x_t\, \kappa_t \tag{7}$$

where $\{\epsilon_t\}$, $\{\kappa_t\}$ and $\{x_t\}$ are independent sequences of random variables.

$\{\epsilon_t\}$ is an independent and identically distributed (iid) sequence of zero-mean Gaussian random variables. We assume that $\mu = 0.00372$ and $\mathrm{Var}\epsilon_t = 0.02^2$ so that $\mu + \epsilon_t$ gives the normally distributed returns of the earlier sections (where the mean arithmetic return was 0.4% per month). $\{\kappa_t\}$ is an iid sequence of Bernoulli random variables with probability distribution $\mathbb{P}\left(\kappa_t = 1\right) = 1 - \mathbb{P}\left(\kappa_t = 0\right) = p$. Here p is small and is the probability of a "rare event", such as a market correction, in a given month. Finally, $\{x_t\}$ is an iid sequence of Gaussian random variables with mean μ_x and standard deviation σ_x. Given that a market correction occurs in month $(t-1,\, t)$, the size of the market correction is represented by x_t.

We repeat the simulations of the earlier section, with unconstrained contributions and with $\lambda_1 = 0.2$, $\lambda_2 = 0.01$ and $i_A = 0.4\%$. Table 6 shows that the volatility and 95th percentile of terminal deficits are lower in the targeted-contribution plan than in the fixed-contribution plan, for various values of p and μ_x, and with $\sigma_x = 0.1$. The targeted-contribution plan is effective at reducing end-wealth risk compared to the fixed-contribution

		Mean deficit		St. deviation of deficit		95th percentile of deficit	
		FC	*TC*	*FC*	*TC*	*FC*	*TC*
$p = 0$	(no jump)	-0.036	-0.003	9.458	3.195	14.591	5.155
$p = 1/60$	$\mu_x =$ -7%	3.234	0.203	11.210	3.838	21.414	6.052
$p = 2/60$	$\mu_x =$ -7%	6.381	0.342	12.450	4.407	26.322	7.323
$p = 3/60$	$\mu_x =$ -7%	9.273	0.502	13.394	4.870	31.183	8.990
$p = 1/60$	$\mu_x =$ -7%	3.234	0.203	11.210	3.838	21.414	6.052
$p = 1/60$	$\mu_x =$ -10%	4.629	0.282	11.509	4.039	23.750	6.437
$p = 1/60$	$\mu_x =$ -13%	5.958	0.358	11.963	4.296	25.889	6.916

Table 6: Mean, standard deviation and 95th percentile of terminal deficits under fixed-contribution plan (FC) and targeted-contribution plan (TC), for various parameters of discrete-time jumps in asset returns, with $\sigma_x = 0.1$, $\lambda_1 = 0.2$, $\lambda_2 = 0.01$, $i_A = 0.4\%$.

plan, even when asset returns have fat-tailed non-normal distributions.

In Table 6, $p = 1/60$ means that on average one jump occurs in a 5-year period. As p increases, market corrections become more frequent. As μ_x becomes more negative, the severity of downward market corrections increases. Table 6 shows that, if a financial planner does not allow for jumps in asset returns, both the fixed and targeted contribution plans will experience greater end-wealth risk as either the frequency or the severity of market corrections increases. However, end-wealth risk remains lower in the targeted-contribution plan compared to the fixed-contribution plan.

Next, we consider investment returns that are serially correlated (and normally distributed), as might be the case if the fund is invested in cash and fixed-income securities. We assume that the log-return δ_t on the fund in month $(t-1, t)$ follows an autoregressive process of order 1, AR(1):

$$\delta_t - \mu \ = \ \alpha(\delta_{t-1} - \mu) + \epsilon_t \tag{8}$$

where $|\alpha| < 1$ and $\{\epsilon_t\}$ and μ are as defined earlier (equation (7)). When $\alpha = 0$, this gives serially independent returns with a mean arithmetic return of 0.4% per month.

Table 7 shows that the volatility and 95th percentile of terminal deficits are smaller in the targeted-contribution plan than in the fixed-contribution plan, for various values of parameter α. The lag-1 autocorrelation in $\{\delta_t\}$ increases as the autoregressive parameter α increases. (The variance of $\{\delta_t\}$ also varies with α.)

The targeted-contribution plan therefore appears to be less risky than the fixed-contribution plan, even with investment returns that exhibit non-normality and serial correlation.

It is also worth highlighting that the asset allocation decision has been ignored in this paper, for simplicity. As set out in the Introduction, we assume that the individual is saving for a medium term of 5 years in a vehicle such as a variable-rate savings bank account or in a low-risk mutual fund. Asset allocation becomes more important with longer-term savings, and this is explored in detail by Campbell and Viceira (2002) among

α	Mean deficit		St. deviation of deficit		95th percentile of deficit	
	FC	*TC*	*FC*	*TC*	*FC*	*TC*
-0.6	0.341	0.014	6.000	2.385	9.795	3.875
-0.4	0.270	0.011	6.795	2.516	10.998	4.070
-0.2	0.155	0.060	7.894	2.784	12.436	4.503
0	-0.036	-0.003	9.458	3.195	14.591	5.155
0.2	-0.386	-0.018	11.836	3.814	17.663	6.074
0.4	-1.135	-0.049	15.882	4.797	22.338	7.539
0.6	-3.246	-0.120	25.425	6.555	30.232	10.039

Table 7: Mean, standard deviation and 95th percentile of terminal deficits under fixed-contribution plan (FC) and targeted-contribution plan (TC), for various values of AR(1) parameter α. $\lambda_1 = 0.2$, $\lambda_2 = 0.01$, $i_A = 0.4\%$.

others. Asset allocation depends on the individual's coefficient of risk aversion and elasticity of intertemporal substitution of consumption, as well as on his labour income risk and investment horizon, and on issues such as taxation and transaction costs etc. In the insurance and actuarial literature, asset allocation is discussed by Vigna & Haberman (2001), Taylor (2002), Owadally & Haberman (2004), Battocchio & Menoncin (2004), Cairns, Blake and Dowd (2006), Emms & Haberman (2008) among others.

The asset allocation decision can be incorporated in future research. It is possible to model lifestyling portfolios, such as life-cycle or target-date funds (Shiller, 2005; Blake, Cairns and Dowd, 2001). The method employed to adjust contributions can also be used to adjust asset allocation, as in Gandolfi, Sabatini and Rossolini (2007).

An Application to Pension Funding

In this section, we consider a long investment horizon and we use a bootstrap stochastic asset model resampled over 59 years of equity and bond return data. We implement the targeted-contribution method in the funding of a defined-benefit pension plan and investigate whether it leads to a better long-term funding position compared to a typical actuarial funding method. It is assumed that an employer sponsors the pension plan and pays contributions into a pension fund to provide retirement benefits to its employees.

We use the model pension plan described by Owadally (2003), except that he uses deterministic economic scenarios whereas we use a stochastic asset model. A key assumption is that employees' salaries, and also their pensions in retirement, grow in line with price inflation. All the monetary quantities that appear below are therefore be expressed in real terms, i.e. net of price inflation.

Pension liabilities have a simplified structure. Employees join at the age of 20. The age profile of the pension plan membership is constant, as is the number of new entrants to the plan every year. A pension, indexed with price inflation, is paid when employees retire at the age of 65. Mortality follows a standard actuarial life table (English Life Table No. 12 for males). Payroll is constant in real terms and the real yearly benefit outgo is normalized to 1. The actuarial liability is calculated at 16.94 (in real terms) using the projected unit credit method and a real annual discount rate of 4% (McGill *et al.*, 2004, p. 665).

On the asset side, the pension fund is assumed to be invested in UK equities and long-term Treasury bonds (gilts) only. We use the bootstrap method to simulate real returns on these assets (Efron and Tibshirani, 1993). We resample real annual returns from the equity and gilt indices calculated by Barclays Capital (2009) from 1950 to 2008, with income reinvested. The returns are net of price inflation, as given by the cost of living index also calculated by Barclays Capital (2009). The pension fund portfolio is rebalanced every year and a 60:40 equity:gilt ratio is maintained.

Annual actuarial valuations take place and the sponsor's contributions are varied to make up any deficit in the plan. The deficit, also known as an unfunded liability, is the excess of the actuarial liability over assets, and a surplus is just a negative deficit. We assume that there is no deficit initially, for example from setting up the plan or from recent improvements to employee benefits. (That is, the initial unfunded liability is zero. Alternatively, any initial unfunded liability could be amortized by means of a separate schedule of payments from the plan sponsor. See for example McGill *et al.* (2004, p. 620).)

The pension plan actuary calculates actuarial gains and losses at yearly valuations of the pension plan. A gain (loss) occurs if experience is more (less) favourable than was assumed at the previous valuation (McGill *et al.*, 2004, p. 621). We assume here that the only source of gains and losses is the investment performance of the pension fund. That is, all other economic and demographic factors turn out to be as assumed.

The typical actuarial practice is to amortize investment gains or losses over 5 years (McGill *et al.*, 2004, p. 686), which is also assumed here. If investment returns are consistently lower than the investment return assumption i_A made by the actuary at each actuarial valuation, then investment losses will occur and will build up into a deficit. Higher employer contributions are then required as these losses are amortized. Our aim is to compare the amortization method with the targeted-contribution method given by equation (4).

Figure 3 shows the distribution of deficits after 40 years when the investment return assumption i_A overstates the long-term average real return on the fund by 3% per annum. (Using the data from Barclays Capital (2009), the pension fund makes an average annual real return of 6.15%, so $i_A = 9.15\%$ here. Parameters λ_1 and λ_2 in equation (4) are equal to 0.5 and 0.03 respectively. A 40-year horizon is appropriate because employees typically have a 40–50 year working lifetime.) The spread of deficits, and the probability of large deficits, appear to be smaller when the targeted-contribution method is used than when amortization is used.

Table 8 displays various statistics for the deficits under both the amortization and the targeted-contribution method, for different errors in the investment return assumption i_A. The standard deviation and 95th percentile of the deficit show that the targeted-contribution method is less risky than the amortization method in the sense that large deficits are less likely under the former. The targeted-contribution method is also less sensitive to errors in the investment return assumption, as the 95th percentile and standard deviation do not change considerably for different investment return assumptions. This mirrors the results of section .

Various commentators (e.g. Watson Wyatt, 2009) have noted a shift from equities to bonds in pension fund asset allocation over the past few years. We repeated the simulations above for equity:gilt allocations of 50:50 and 40:60. The conclusions are essentially unchanged, i.e. the targeted-contribution method results in smaller deficits and is less sensitive to the return assumption.

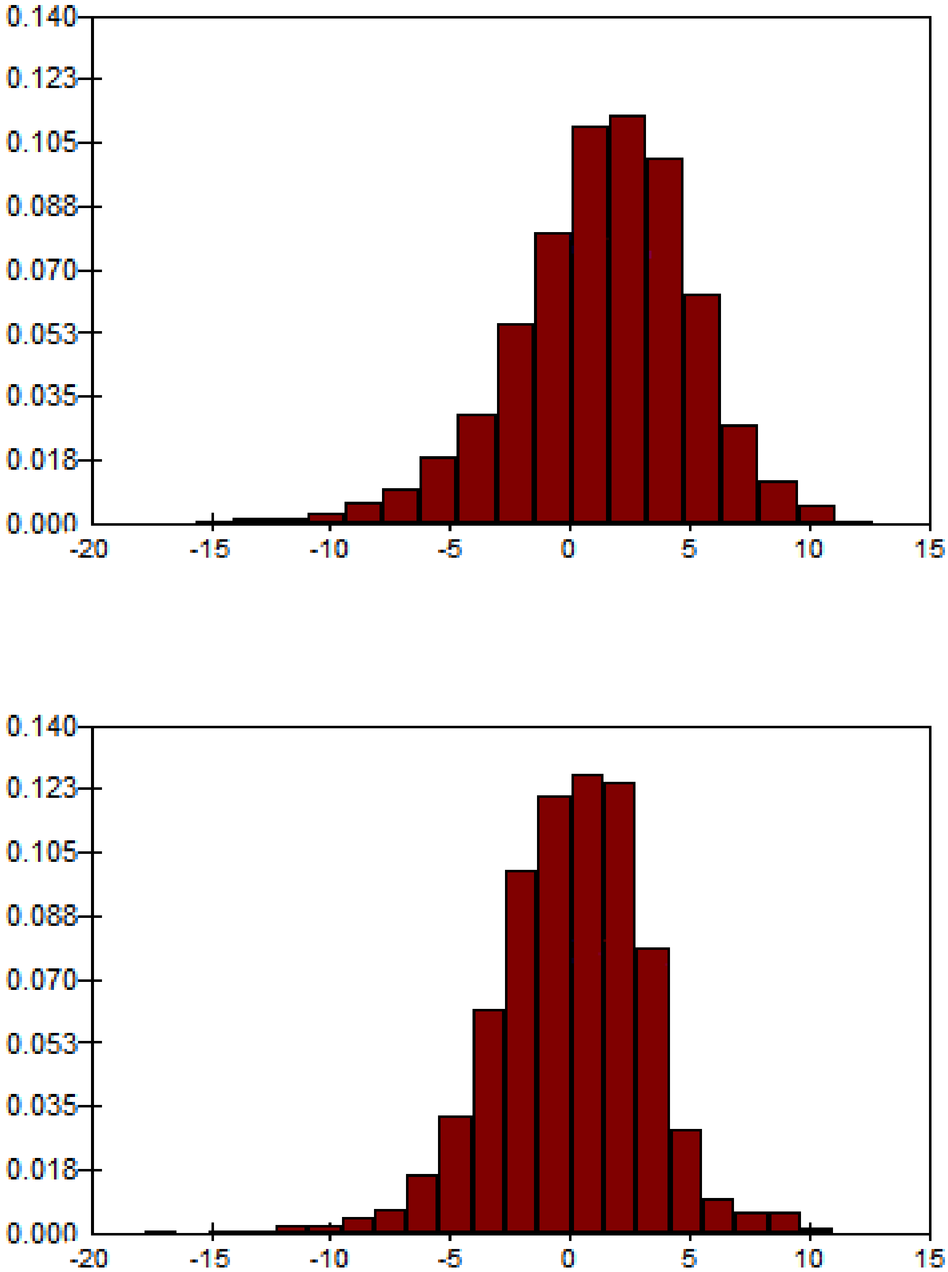

Figure 3: Histograms with identical scales for the deficit after 40 years in a pension plan with amortization (above) and with targeted contributions (below).

Error in i_A	Mean deficit		St. deviation of deficit		95th percentile of deficit		Mean square deficit	
	A	TC	A	TC	A	TC	A	TC
-3%	-1.607	-0.069	4.673	3.257	5.186	4.537	24.414	10.613
-2%	-1.046	-0.048	4.503	3.252	5.515	4.551	21.370	10.579
-1%	-0.512	-0.027	4.342	3.248	5.826	4.565	19.118	10.547
0	-0.003	-0.007	4.190	3.243	6.115	4.579	17.559	10.516
1%	0.483	0.014	4.046	3.238	6.420	4.593	16.605	10.487
2%	0.948	0.033	3.910	3.234	6.671	4.606	16.182	10.459
3%	1.391	0.053	3.780	3.229	6.937	4.619	16.222	10.432

Table 8: Mean, standard deviation, mean square and 95th percentile of deficits in pension plan after 40 years with amortization (A) and with targeted contributions (TC).

The average values of the pension fund assets and contribution over 40 years are shown in Table 9 for a 40:60 equity:gilt portfolio, which has a long-term average real return of 4.82%, and a real return assumption i_A of 5.82% (so that the error in i_A is 1%). Because the investment return assumption i_A is too optimistic, both the amortization and targeted-contribution methods start off with a low contribution from the employer initially. As deficits mount in the pension fund, the targeted-contribution method demands larger contributions more quickly than the amortization method. After 40 years, the fund levels off at an average value of 16.92 in the targeted-contribution case and 16.45 in the amortization case, compared to the actuarial liability value of 16.94. The average deficit is therefore almost zero with the targeted-contribution method, whereas it is never completely removed under amortization and a higher contribution is permanently required on average.

Note that under both methods, the pension fund is in balance after 40 years. In the

time	Mean fund		Mean contribution	
(years)	A	TC	A	TC
0	16.9364	16.9364	0.0680	0.0680
1	16.7734	16.7734	0.1044	0.1544
2	16.6463	16.6964	0.1393	0.2001
3	16.5433	16.6597	0.1753	0.2268
4	16.4751	16.6521	0.2108	0.2391
5	16.4374	16.6539	0.2469	0.2467
6	16.4485	16.6731	0.2438	0.2450
7	16.4478	16.6844	0.2443	0.2469
8	16.4393	16.6924	0.2455	0.2502
9	16.4497	16.7183	0.2432	0.2438
10	16.4503	16.7320	0.2419	0.2431
15	16.4348	16.7877	0.2471	0.2407
20	16.4433	16.8390	0.2456	0.2328
25	16.4576	16.8750	0.2405	0.2261
30	16.4588	16.8939	0.2378	0.2240
35	16.4454	16.9007	0.2420	0.2262
40	16.4541	16.9194	0.2423	0.2209

Table 9: Mean values of fund and contribution over time in a pension plan with amortization (A) and a plan with targeted contributions (TC).

amortization case, a fund of 16.45, with contribution income of 0.24 and benefit outgo of 1, accumulates at an investment return of 4.82% to 16.45 again after a year. In the targeted-contribution case, a fund of 16.91, with contribution income of 0.22 and benefit outgo of 1, accumulates at an investment return of 4.82% to 16.91 again after a year. However, the pension fund is in balance with a nearly zero average deficit in the latter case.

On the Barclays Capital (2009) data, autocorrelations on real equity returns up to a lag of 10 years were not significant at 5%. Autocorrelations on real gilt returns up to lag 10 were not significant at 5%, except at lag 5. This appears to have little physical meaning. Nevertheless, we also incorporated serial correlation in the simulations by resampling using blocks of 3 years and 5 years (Efron and Tibshirani, 1993). Again, the qualitative conclusions above are unchanged. (The detailed results are not included here to save space and are available from the authors.)

Conclusion

We considered a savings plan where contributions are received and invested for the purpose of meeting a given liability in the future. An example could be an investment and savings vehicle being used to meet school or college fees in 5 years' time. In practice, financial advisers recommend a fixed and level regular contribution, based on an assumed investment return on the fund. Since investment returns are random and will be different from the assumed investment return, the fund usually undershoots or overshoots the target fund objective, leading to a deficit or surplus respectively.

We investigated an alternative savings plan, based on a method borrowed from industrial process control and econophysics. Under this method, contributions are flexible and are adjusted and targeted systematically so that the fund objective is met. We used stochastic simulations to measure the risk that a terminal deficit occurs, that is, that end-of-period wealth falls short of the target fund. The targeted-contribution method appears to be less risky than the fixed-contribution method, in the sense that the volatility and 95th percentile of the deficit are smaller in the former case. In particular, the size of deficits in the targeted-contribution plan is less sensitive to errors in the financial planner's estimate of future investment return, than in the fixed-contribution plan. These results appear to be robust under a wide range of parameter values, with and without constraints on contributions, with asset returns that are serially correlated, and with returns that incorporate large but rare market corrections and exhibit non-normality. Finally, we applied the targeted-contribution method to a defined-benefit pension plan, with a long investment horizon and using bootstrap resampling from actual equity and bond market data. The standard deviation, mean square and 95th percentile of the deficit in the pension fund were again lower with the targeted-contribution funding method than with a conventional pension funding method.

A practical implication of this research for financial planners is that savings plans can be designed that are more flexible for individuals. A suitable design, such as the one described in this paper, should be not only flexible but also targeted in a systematic way to achieve robust outcomes for individual investors.

This research can be extended in several directions, as discussed in the paper. First, the asset allocation decision should be explored. Lifestyling portfolios can be readily included (Shiller, 2005). The method employed to adjust contributions can also be used to adjust asset allocation, as in Gandolfi, Sabatini and Rossolini (2007). Secondly, the investment objective considered in this paper was simple and related to the end-of-period wealth only. If the targeted-contribution plan is used for longer-term savings, such as retirement plans, then the utility of both wealth at retirement and consumption during the accumulation phase must be considered. Finally, the distribution of the deficit and the evolution of contributions in the targeted-contribution method are also of interest. Further simulations, with back-testing on historical data, should help with the choice of parameters.

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