

Optimal Time for Disinvestment and Acquisition in the Bivariate Compound Poisson model

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Abstract

Let us assume that the uncontrolled joint surplus (X_t^1, X_t^2) of two branches of an insurance company follows a bivariate compound Poisson model:

$$\begin{aligned} X_t^1 &= x_1 + p_1 t - \sum_{i=1}^{N_t^1} U_i^1 - \sum_{j=1}^{N_t^3} \gamma_1 U_j^3 \\ X_t^2 &= x_2 + p_2 t - \sum_{i=1}^{N_t^2} U_i^2 - \sum_{j=1}^{N_t^3} \gamma_2 U_j^3, \end{aligned}$$

where p_1 and p_2 are the premium rates of the branches, N_t^1, N_t^2 and N_t^3 are independent Poisson processes with intensities β_1 , β_2 and β_3 . We assume that U_i^j are i.i.d random variables with distribution function F_j and that U_i^1, U_i^2, U_i^3 are mutually independent and also independent of the Poisson processes. This model reflects that could be dependence between some claims of the two branches.

The manager has the possibility of disinvesting one of the two branches at any time τ paying fees $c_1, c_2 \geq 0$ depending on which branch is disinvested. We also consider the case of negative c_1 and c_2 , meaning that the company sells one of the branches with no reserve at price c_i .

The company pays dividends to the shareholders. The goal is to find both the disinvestment time and the dividend payment strategy which maximize the expected discounted dividend payment up to the ruin time. Before the disinvestment time, the ruin occurs when the sum of the surpluses of both branches becomes negative; after the disinvestment time, the ruin occurs when the surplus of the surviving branch becomes negative. This optimal stopping problem is equivalent to a first-order integro-differential obstacle problem involving a free boundary that corresponds to the disinvestment decision.

We characterize the value function of this optimization problem as the smallest viscosity supersolution of the associated Hamilton-Jacobi-Bellman equation with obstacle. We also obtain the optimal time and the optimal dividend payment strategy for some claim-size distributions.

We also study the dual problem of optimal time for acquisition.

Keywords: bivariate compound Poisson model, acquisition, disinvestment, stopping time, optimal dividend payment strategies.