STRUCTURE OF SUBCLASSES IN THE FRAME OF HEAVY-TAILED DISTRIBUTIONS

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Abstract. The structure of the heavy tails has been studied extensively. The most known result is given by figure [4, Fig. 1.4.1], which describes the relationship between the main classes of distributions such as long tails, subexponentials tails , dominatedly varying tails, regular varying tails and slowly varying tails. As it custom we denote them as $\mathcal{L}$, $\mathcal{S}$, $\mathcal{D}$, $\mathcal{R}_{-a}$, $a > 0$ and $\mathcal{R}_0$. It's common truth that more classes have been introduce to support the demands in different areas of applied probability such as risk theory. As a result plenty counterexamples have been given to sketch the complicate structure.

On the other hand the distribution of main interest and not the designed examples, can be presented in a more simple relation. Roughly speaking, the subexponential class can be divided into two disjoint parts as

$$\mathcal{S} \approx (\mathcal{D} \cap \mathcal{L}) \cup (\mathcal{S} \cap \mathcal{R}_{-\infty}), \tag{0.1}$$

where $\mathcal{R}_{-\infty}$ is the class of rapidly varying tails (see Definition [4, A3.11]).

We will present two commonly fulfilled assumptions. All the distributions that fulfill this assumptions not only satisfy relation (0.1) as equality, but also the structure of classes of the extended rapidly varying, slowly varying and constantly varying tails (see Definitions[1,], [4, A3.1] and [3, p.118]) is given in an also simple relation. The results are based on the relation between hazard rates functions and distributions,which is a well known fact see for instance [5], and the aid of Matuszewska indices (see [2]).

References

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