Local risk-minimization with longevity bonds

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Extended abstract: The study of mortality derivatives has increased in popularity during the last years. The motivation is the increasing average lifetime in many countries which exposes the life insurance companies to a major non-diversifiable risk. In particular, the increased popularity is caused by the forthcoming Solvency II legislation. Based on the results of QIS V, see EIOPA Report on the fifth Quantitative Impact Study (QIS5) for Solvency II, one realises that the longevity stress is the second largest risk of the life module. Hedging this risk by means of longevity derivatives could potentially reduce the solvency capital requirement. The longevity risk comes along with the financial risk. One challenge for the life insurance companies in the coming years will be to control or minimize the combined risk. A possible way to do this is to hedge the risk by trading mortality derivatives. This has to a limited degree been implemented along with the introduction of mortality derivatives in the financial markets the last few years. The market for longevity derivatives is right now evolving as shown in the conference summary from "Longevity 7: Seventh International Longevity Risk and Capital Markets Solutions Conference".

This research studies the hedging criterion of local risk-minimization for life insurance contracts in an incomplete financial market which includes longevity bonds. The longevity bond is a bond that specifies payments which are defined as the current number of survivors in a given portfolio of insured lives. The number of survivors is modeled via a double-stochastic process, where the mortality intensity is driven by a time-inhomogeneous CIR-model. In addition to the longevity bond, the financial market is assumed to consist of a traditional bond and a savings account. The interest rate is driven by a standard Vasiček model. We define the price process of the longevity bond by introducing a pricing measure. The paper extends the results of Dahl, Melchior and Møller (2008) to the case where the traded assets are not martingales under the measure used for determining the optimal hedging strategies. In this case, one can not apply the criterion of risk-minimization as done in Dahl et al. (2008). Instead we apply the criterion of local risk-minimization within the model of Dahl et al. (2008). If the price processes of the assets are continuous, the criterion of local risk-minimization is equivalent to the criterion of risk minimization under the minimal martingale measure. We can not benefit from this, since the continuity-condition is violated by the jumps in the underlying counting process, which generate jumps in the price process of the longevity bond. Instead, we put up

some conditions for the strategy. The conditions lead to the locally risk-minimizing strategy.

Here, we give a short review of the literature on subjects considered in the paper. Risk-minimization was introduced by Föllmer and Sondermann (1986) and the criterion has been used in several papers subsequently. Considering life insurance, Dahl et al. (2008) carries out risk-minimization for insurance payment streams for different markets in the case where the interest rate process and the mortality intensity process are continuous. The theory of local risk-minimization was introduced in Schweizer (1988). Schweizer (2008) gives a streamlined theoretical description of local risk-minimization for multidimensional payment processes, but does not provide tools for finding local risk-minimizing strategies. The criterion of local risk-minimization has recently been used by various authors in insurance applications. An example of the theory used in a setting with jumps is Vandaele and Vanmaele (2008). They determine locally risk-minimizing strategies for an insurance contract in a market with a risky asset containing jumps and a deterministic interest rate.

The research is organized as follows: We introduce a financial market with zero coupon bonds, a model for the mortality intensity and a portfolio of insured lives. Both the interest rate and the mortality intensity are modeled as in Dahl et al. (2008). We present the combined model for the financial market and the portfolio of insured lives and study a class of equivalent martingale measures. We examine mortality derivatives in the form of longevity bonds and a general insurance contract and find their stochastic representations. We determine the minimal martingale measure for the market and we find conditions which ensure that the minimal martingale measure is a probability measure. We finally reach to the main result: The locally risk-minimizing hedging strategy for the insurance contract. This strategy is compared to the risk-minimizing hedging strategy under the minimal martingale measure.

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