Multivariate generalized Cox processes and tail conditional expectations

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Let $N(t) = (N_1(t), \ldots, N_m(t))$ be a multivariate Poisson processc with independent standard components. Consider also random process $\Lambda(t) = (\Lambda_1(t), \ldots, \Lambda_m(t))^T, t \geq 0$, whose trajectories begin from origin, nondecreasing and right continuous. Besides that suppose that $E(\Lambda_k(t)) = l_k \cdot t$, $cov(\Lambda_j(t), \Lambda_k(t)) = S_{jk} \cdot t$, $D(\Lambda_k(t)) = S_{kk} \cdot t = S_k^2 \cdot t$, where $l_k > 0$, $S = (S_{jk})$ – positive definite matrix. Let $X_i = (X_{i1}, \ldots, X_{im})^T$ be a sequence of i.i.d. random vectors such that $E(X_j) = a = (a_1, \ldots, a_m)^T$, $cov(X_{ij}, X_{ik}) = \sigma_{jk}$, where $a_k \neq 0$, $\Sigma = (\sigma_{jk})$ – positive definite matrix. Processes $(N(t), t \geq 0)$, $(\Lambda(t), t \geq 0)$ and sequence (X_i) are independent.

We consider the following variant of multivariate generalized Cox process: $C(t) = (C_1(t), \ldots, C_m(t))$:

$$C_k(t) := \sum_{j=1}^{N_k(\Lambda_k(t))} X_{jk} \ .$$

We are interested in asymptotic behaviour of process $C(t)$ as $t \to \infty$ and propose a necessary and sufficient condition for convergence of the distribution of $C(t)$ with nonrandom centering and normalization to shift mixture of multivariate normal distribution as $t \to \infty$.

Theorem. *The convergence*

$$\frac{C(t) - A(t)}{\sqrt{t}} \Rightarrow Z \ , t \to \infty \ , \tag{1}$$

holds for some random vector Z, where $A_k(t) = a_k \cdot l_k \cdot t$, if and only if the convergence

$$\frac{\Lambda(t) - \tilde{A}(t)}{\sqrt{t}} \Rightarrow V \ , t \to \infty \ , \tag{2}$$

holds for some random vector V, where $\tilde{A}_k(t) = l_k \cdot t$. Moreover

$$Z_k \stackrel{d}{=} \sqrt{l_k(\sigma_k^2 + a_k^2)} \cdot W_k + a_k \cdot V_k \ , \tag{3}$$

where $W = (W_1, \ldots, W_m)$ are independent random variables with standard normal distributions and W and V are independent.

Our result is the analog of the result from [1].

Some applications to actuarial and financial mathematics are considered. For example, using our result and the methods from papers [2] and [3] we can calculate asymptotically tail conditional expectations.

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References

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