Modeling Joint Extreme Events Using Multivariate Self-Exciting Jump Processes

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We study the clustering behaviour of multivariate extreme events in the context of a peaks-over-threshold (POT) approach to modelling of extremes. We concentrate on the typical features observed in real data: (i) the distribution of both the intervals between extreme events and their magnitudes depends strongly on the past; (ii) both individual and joint extremes influence the arrival of future extreme events; (iii) the dependence between individual extremes varies significantly over time. We develop an approach to model the probability of multivariate joint extreme events and demonstrate that all these features can be captured in an efficient way. In particular, joint extreme events are modelled as a non-homogeneous Poisson process with a conditional intensity that incorporates the effect of past marginal and joint extremes using the self-exciting POT model with predictable marks for modelling individual extremes, and an extreme value copula for modelling the dependence structure. The dependence parameter of the copula is allowed to depend on a self-exciting process in order to capture changes in the interdependencies between extreme events. Since the model is based on the asymptotic arguments of multivariate extreme value theory it can provide the conditional likelihood of joint extreme events for any sufficiently high threshold. The model allows also for an asymmetric effect of individual extremes on the arrival of joint extreme events.
We discuss the properties of this model, treat its estimation, deal with testing the goodness-of-fit, and develop an algorithm to simulate artificial data from the proposed model. As an application, we analyze joint extreme events of the USA and European financial markets and joint extreme events in European banking sector considering four major European banks. The empirical results demonstrate a good fit of the model and suggest the empirical importance of the self-exciting feature for modeling both marginal and joint extreme events and for capturing changes in the interdependencies between them.
Keywords: Time Series, Hawkes processes, Peaks over Threshold, Extreme Value Copula.

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