

Valuation and Risk Assessment of Disability Insurance using a Discrete Time Non-homogeneous Backward Semi-Markov Reward Process

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During the last decades, the importance and demand for disability insurance has continuously risen, which allows the application of new methods of premium calculation due to an increased data basis. The previously used formulae, e.g. the Norwegian model or the Manchester Unity model, can now be replaced by semi-Markov reward processes in order to provide a sound framework not only for premium calculation but also for risk analysis. This constitutes a major issue for life insurers in order to more adequately assess the value and risk associated with these contracts, which is especially relevant against the background of Solvency II. However, even though disability insurance and dread-disease products are becoming increasingly important especially in the German market, there has been only very little research on this topic to date.

In this paper, we extend previous literature by adding permanence and instant rewards to a time discrete, non-homogeneous semi-Markov process with initial and final backward recurrence time processes as introduced by D'Amico, Guillen and Manca (2009). The insertion of rewards into the model presented by D'Amico et al. (2009) allows us to more precisely evaluate a disability insurance policy on account of the consideration of major drivers in disability insurance, which has not been done so far. Thus, our model incorporates all rewards paid such as premiums and benefits as well as the duration since the time of disablement, which has a considerable impact on disability probabilities, mortality rates, durations and reactivation (Seegerer, 1993). We further provide a closed-form solution and a simulation approach for computing all higher moments of the proposed model that are needed for a thorough risk analysis of disability insurance contracts.

Based on assumptions relying on empirical results from D'Amico et al. (2009), higher moments for multiple risk groups are analyzed in combination with different premiums paid by the policyholder. Additionally, the influence of the waiting time distribution for each state transition on the risk inherent to the insurance contract is examined. Finally, we compare our results with those obtained by the model of Stenberg, Manca and Silvestrov (2007) and with those acquired by the actuarial approach as laid out in Haberman and Pitacco (1999). Our analysis indicates a considerable difference between the model of Stenberg et al. (2007), the actuarial approach and our approach. Since all models are ascertained to be based upon identical assumptions, the inclusion of the duration effect for disabled policyholders is the sole driving force. One main result is that regardless of the assumed waiting time distribution, the duration effect is a significant feature, which should be implemented in disability models in order to adequately map the risk. Overall, our new approach allows new insight with respect to the assessment of an insurer's risk situation.

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Duration model, multi state model, reward process, higher moments

Main references:

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