CDO Valuation Using Stochastic Correlations in Structural Models

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We study structural models in order to valuate tranches of collaterized debt obligations, CDO-s. The values $X_t^{(i)}$ of individual companies, the "names", may follow Markov processes, eventually with jumps, but to keep matters simple for demonstrative purposes we only consider Ornstein–Uhlenbeck (O-U) processes here as special cases:

$$dX_t^{(i)} = -aX_t^{(i)}dt + dB_t^{(i)};$$

The company defaults if its value goes below a barrier, that is calculated for a given single name from its survival probability curve, inferred eventually from market quotes of CDS written on that name. Given the survival probabilities as input, the barrier curve can be computed by solving the inverse passage problem.

In valuing CDO tranches the joint defaults play essential role, hence, the values of names must be furnished with a suitable interdependence structure. The one we specify varies the joint distribution only while keeps marginals fixed as O-U processes with unchanged parameter, so, survival probabilities remain unchanged. To achieve that we build up $B_t^{(i)}$-s as interdependent Wiener processes by modifying the simple one factor model. In the latter the driving Wiener processes admit a decomposition with a common Wiener process Z_t and idiosyncratic ones $\epsilon_t^{(i)}$, expressed as

$$dB_t^{(i)} = \rho dZ_t + \sqrt{1-\rho^2}d\epsilon_t^{(i)}. \tag{1}$$

Here the constant ρ can be interpreted as correlation among the Wiener components. To create a more complex non-linear interdependence structure we change it to be a random process ρ_t that we call stochastic correlation. ρ_t is defined conditionally on the common factor Z_t. When ρ_t is adapted and predictable wrt. the filtration of Z_t, and the idiosynchratic components are independent of Z_t and of each other, Lévy's characterisation theorem yields that all $B_t^{(i)}$-s are Wiener processes, resulting in intact marginal distributions for $X_t^{(i)}$-s.

We suggest to create the stochastic correlation ρ_t either by functional dependence on the incremets of the factor, or by a Jacobi process driven by the factor. In either way the joint distribution of $B_t^{(i)}$-s at fixed time t ceases to be multidimensional Gaussian, furthermore formal tests reject the created copula to belong to known copula families.

We evaluate the obtained tranche spreads in terms of the base correlation curve. Both simulated artificial and traded real sample portfolio names endowed with the stochastic correlation resulted in mostly concave base correlation curves. Exactly that can intuitively be expected when tail dependence is significant and that can be observed also in real trading situations. Stochastic correlation is flexible in creating great variety of base correlations, and transparently controllable by a couple of parameters.

Keywords: Base correlation, CDO valuation, credit risk, mathematical finance, stochastic correlation