Indirect Estimation of R-GARCH Models

Pedro A. Morettin and Jhames M. Sampaio

University of São Paulo, Brazil

1 Introduction

Many studies in finance indicate the necessity of models to describe the heavy tails that financial data present. The class of Randomized GARCH models, proposed by Nowicka (1998) allows the use of stable innovations in the usual GARCH models. This paper suggests the use of indirect inference (Gouriéroux et al., 1993) to estimate R-GARCH models. As the autocovariance function does not exist in general for this class of processes, another measure of dependence has to be used and this is also a focus of this paper.

2 R-GARCH models

An R-GARCH(r,p,q) model, r,p,q $\in \mathcal{N}$ is defined by

$$r_t = \sqrt{h_t}\epsilon_t, \; t = 0, \pm 1, \pm 2, \cdots, \quad (1)$$

$$h_t = \sum_{i=1}^{r} \theta_i \eta_{t-i} + \sum_{j=1}^{p} \phi_j h_{t-j} + \sum_{k=1}^{q} \psi_k r_{t-k}^2, \quad (2)$$

where

$$r \geq 1, \theta_r > 0, \theta_i \geq 0, i = 1, \cdots, r-1, \quad (3)$$

$$p \geq 0, \phi_p > 0, \phi_j \geq 0, j = 1, \cdots, p-1, \quad (4)$$

$$q \geq 0, \psi_q > 0, \psi_k \geq 0, k = 1, \cdots, q-1, \quad (5)$$

the r.v.'s ϵ_t are i.i.d. $\sim \mathcal{N}(0,1))$, the r.v.'s η_t are positive i.i.d., $\{\epsilon_t\}$ and $\{\eta_t\}$ are independent.

We will consider the η_t's strictly stable, with

$$\eta_t \sim S_{\alpha/2}\left(2\left(\cos\frac{\pi\alpha}{4}\right)^{2/\alpha}, 1, 0\right), \quad (6)$$

where $0 < \alpha < 2$.

3 Indirect Inference

Applies when the direct estimation (MLE) is difficult and simulation is easy. We have a model of interest and an auxiliary model.

- (Auxiliary model) GARCH(1,1) model with $\epsilon_t \sim$ normal, t-Student:

$$\begin{aligned} r_t &= \sqrt{h_t}\epsilon_t, \quad t = 0, \pm 1, \pm 2, \cdots, \\ h_t &= \alpha_0 + \alpha_1 r_{t-i}^2 + \beta_1 h_{t-j}, \end{aligned}$$

  where $\alpha_0 > 0$ and $(\alpha_1 + \beta_1) < 1$.

- (Original model) R-GARCH(1,1,0) model:

$$\begin{aligned} r_t &= \sqrt{h_t}\epsilon_t, \quad t = 0, \pm 1, \pm 2, \cdots, \\ h_t &= \theta_1 \eta_{t-1} + \phi_1 h_{t-1}, \end{aligned}$$

  where ϵ_t are i.i.d. standadrd normal,t-Student or α-stable, η_t are i.i.d. according to (6).

4 Dependence structure

The autocovariance function is not defined for stationary time series, α-stable, when $\alpha < 2$, hence we need another way to assess dependence. The tool here is the autocovariation function

$$I_k(\xi_1, \xi_2) = -\ln \mathbf{E}\left[e^{i(\xi_1 X_t + \xi_2 X_{t-k})}\right] + \ln \mathbf{E}\left[e^{i\xi_1 X_t}\right] + \ln \mathbf{E}\left[e^{i\xi_2 X_{t-k}}\right]. \quad (7)$$

We will discuss properties of this function for R-GARCH models and how to estimate it.

5 Simulations and applications

Simulations are given to show the effectiveness of the method and the methodology is applied to two high frequency Brazilian financial series

6 References

1. Gouriéroux, C., Monfort, A. and Renault, E. (1993) Indirect inference. *Journal of Applied Econometrics*, **8**, 85-118.

2. Nowicka, J. (1998). Analysis of measures of dependence for time series with α-stable innovations. Ph.D. dissertation, Wroclaw University of Technology.