Archimedean copulas derived from utility functions

Jaap Spreeuw
Cass Business School, City University London.
E-mail: j.spreeuw@city.ac.uk

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Archimedean copulas are constructed using a one-dimensional function, the additive generator, which is strictly decreasing and convex. Von Neumann-Morgenstern utility functions, on the other hand, are nondecreasing (decision makers prefer more to less) and concave (decision makers are risk averse). Therefore, an affine transformation of a utility function, with sign changed, could act as a generator for an Archimedean copula, subject to some additional conditions.

This class of generators was launched in Spreeuw (2010), who also discussed its basic properties, including the range of dependence covered. Using the results from Avérous and Dortet-Bernadet (2004), it shows how properties of a utility function translate into the type of dependence induced by the Archimedean copula generated from it. This includes notions of positive dependence such as PQD (Positive Quadrant Dependence), LTD/RTI (Left Tail Increasing/Right Tail Decreasing) and SI (Stochastically Increasing). In particular, the relationship between the properties of SI for Archimedean copulas and Decreasing Absolute Risk Aversion (DARA) for utility functions is explored. In economics and decision theory, the notion of DARA, introduced in Pratt (1964) is generally considered to be a desirable property.

The current paper extends Spreeuw (2010) in several ways. First, the restrictions to be imposed on a utility function will be relaxed a bit, in the sense that it only needs to be defined on an interval $[0, M]$ with $M > 0$ (and not necessarily $[0, \infty)$). Secondly, apart from PQD, LTD/RTI and SI, also PLR (Positive Likelihood Ratio dependence) and its corresponding property in economics are considered. In the third place, we will investigate negative dependence, in particular the negative counterparts of the aforementioned notions of positive dependence (NQD, RTD/LTI, and SD). Finally, Archimedean copulas in more than two dimensions are discussed. It is well known that additional constraints apply to the generator of an Archimedean n-copula for $n > 2$, see McNeil and Nešlehová (2009). An interpretation of the corresponding constraints imposed on utility functions is given, and the notions of positive dependence in higher dimensions as discussed in Müller and Scarsini (2005) in terms of properties of the generator are explored.

Several utility functions which appeared in the economic literature will be considered as examples. While some classes of utility functions are related to common Archimedean families (such as Clayton and Gumbel-Hougaard), this research has led to some generators of Archimedean copulas that, to the best of our knowledge are new.

Keywords: Archimedean copulas, additive generators, types of dependence

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