

# OPTIMAL RISK TRANSFERS IN INSURANCE GROUPS

ALEXANDRU V. ASIMIT<sup>1</sup>      ALEXANDRU M. BADESCU<sup>2</sup>      ANDREAS TSANAKAS<sup>3</sup>

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Insurance groups often comprise a number of distinct legal entities, operating in different territories. The risk exposures of different entities will in general not be perfectly correlated, and thus some group level diversification is observed. Furthermore, risks and assets in the group portfolio are not pooled across entities and hence there are limits to the cross-subsidy, as well as the capital fungibility, within the group. Nonetheless, the risk and capital requirements of individual entities can be reduced, through a web of capital and risk transfer arrangements across entities. The complexity of group risk management has motivated a lively academic literature (Keller (2007); Filipović and Kupper (2008); Gatzert and Schmeiser (2011); Schlütter and Gründl (2011)).

The above literature typically analyzes the impact of intra-group risk transfers of pre-specified type. In contrast, the focus here is on deriving optimal functional forms of risk transfers. For this purpose, we use a formal setting with two legal entities, subject to potentially discrepant regulatory requirements. Optimal risk transfers are chosen such that the risk adjusted value of the group liabilities is minimized, when valuation takes place under the cost-of-capital methodology (Wüthrich *et al*, 2010) underlying regulatory valuation approaches such as the Swiss Solvency Test and Solvency II.

A number of results are proved, providing analytical solutions for the corresponding optimization problems, when the capital requirement for each entity is given either by Value-at-Risk (VaR) or Expected Shortfall (ES) and each entity is subject to a different cost of capital, due to potential differences in taxation or other operating costs. The results bear out the properties of the risk measures used, specifically the Value-at-Risk measure's insensitivity to tail risk beyond the given confidence level. In particular, when one entity is subject to a lighter (VaR-based) regulatory requirement than the

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<sup>1</sup>Corresponding author. Cass Business School, City University London, London EC1Y 8TZ, United Kingdom. E-mail: asimit@city.ac.uk. Tel: +44(0)2070405282. Fax: +44(0)2070408572.

<sup>2</sup>Department of Mathematics and Statistics, University of Calgary, Calgary, Alberta T2N 1N4 , Canada. E-mail: abadescu@math.ucalgary.ca

<sup>3</sup>Cass Business School, City University London, London EC1Y 8TZ, United Kingdom. E-mail: a.tsanakas.1@city.ac.uk

other, it ends up being allocated most of the group's extreme risk exposure, in the form of high (usually infinite) reinsurance layers.

Such incentives ought to trouble regulators tasked with protecting policyholder interests. To further investigate the issue, we focus on the case where the first entity is subject to an ES-based capital requirement, while the second entity, acting as subsidiary solely set up to reinsure the first, holds capital according to VaR. We then show that, in the absence of an allowance for credit risk in capital requirements, the transfer of tail risk to the VaR-regulated entity is detrimental to policyholder welfare as it increases the expected policyholder deficit.

Motivated by these findings, we then consider a situation where the counter-party credit risk arising from the risk transfer is reflected in the ES-based capital requirement of the first entity. A corresponding optimization problem is formulated and its solution shows that incentives for transferring tail risk to the second entity vanish. Moreover, policyholder welfare is restored to pre-transfer levels.

*Keywords and phrases:* Cost of Capital, Expected Shortfall, Insurance Groups, Value-at-Risk.

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