Asymptotic Results for Conditional Measures of Association in the Classical Risk Model

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Abstract. Several asymptotic results are obtained for the conditional measures of association, such as Kendall's tau, Spearman's rho and Pearson product-moment correlation coefficient, where the underlying model is given by the classical risk model with finite horizon. The chosen random variables are the first two order statistics of the claim process and the aggregate claim amount. Many of the results have confirmed the "one-jump" property of the risk model, i.e the joint extreme events for the considered random variables are strongly positive dependent. Non-trivial limits are obtained when the dependence among the first two order statistics is considered. Interestingly, the Pearson product-moment correlation coefficient between the first two order statistics provide alternative procedure to estimate tail index of the individual claim amount, and at the same time may help in deciding whether or not the individual claim amount is heavy tailed.

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1. Short description of regularly varying results

We consider the classical risk model for which the claim sizes $X_i, i = 1, 2, \ldots$ are assumed to be positive iid rv's with common df F, which are independent of the claim arrival process $\{N(t), 0 \leq t \leq T\}$, where T is a fixed horizon of interest. Let $X_T^{(1)} \geq X_T^{(2)}, \ldots$ be the order statistics corresponding to the claim sizes occurring during $[0, T]$. By convention, $X_T^{(k)} = 0$ if $N(T) < k$. The aggregate loss during our horizon is denoted by $S_T := \sum_{i=1}^{N(T)} X_i$, where as usually, $S_T = 0$ if $N_T = 0$. The rv's of interest are S_T, $X_T^{(1)}$ and $X_T^{(2)}$, but sometimes, multiple realisations of the process will be needed to perform our calculations, and the trivariate random vector of interest for the i^{th} realisation will be denoted by $\big(S_{T,i}, X_{T,i}^{(1)}, X_{T,i}^{(2)}\big)$. In order to assess the strength of dependence in between the extreme events arising from this process, the following two conditional Kendall's tau are investigated

$$\begin{aligned}\tau^{+1}(t) \; &:= \; \Pr\Big(\big(S_{T,1}-S_{T,2}\big)\big(X_{T,1}^{(1)}-X_{T,2}^{(1)}\big)>0|X_{T,1}^{(1)},X_{T,2}^{(1)}>t\Big) \\ &\quad -\Pr\Big(\big(S_{T,1}-S_{T,2}\big)\big(X_{T,1}^{(1)}-X_{T,2}^{(1)}\big)<0|X_{T,1}^{(1)},X_{T,2}^{(1)}>t\Big)\end{aligned} \tag{1.1}$$

and

$$\begin{aligned}\tau^{12}(t) \; &:= \; \Pr\Big(\big(X_{T,1}^{(1)}-X_{T,2}^{(1)}\big)\big(X_{T,1}^{(2)}-X_{T,2}^{(2)}\big)>0|X_{T,1}^{(2)},X_{T,2}^{(2)}>t\Big) \\ &\quad -\Pr\Big(\big(X_{T,1}^{(1)}-X_{T,2}^{(1)}\big)\big(X_{T,1}^{(2)}-X_{T,2}^{(2)}\big)<0|X_{T,1}^{(2)},X_{T,2}^{(2)}>t\Big)\end{aligned} \tag{1.2}$$

for large values of t. Similarly, the equivalent Spearman's rho rank correlations of interest are

$$\begin{aligned}\rho_R^{+1}(t) \; &:= \; 3\Pr\Big(\big(S_{T,1}-S_{T,2}\big)\big(X_{T,1}^{(1)}-X_{T,3}^{(1)}\big)>0|X_{T,1}^{(1)},X_{T,2}^{(1)},X_{T,3}^{(2)}>t\Big) \\ &\quad -3\Pr\Big(\big(S_{T,1}-S_{T,2}\big)\big(X_{T,1}^{(1)}-X_{T,3}^{(1)}\big)<0|X_{T,1}^{(1)},X_{T,2}^{(1)},X_{T,3}^{(1)}>t\Big)\end{aligned} \tag{1.3}$$

and

$$\begin{aligned}\rho_R^{12}(t) \; &:= \; 3\Pr\Big(\big(X_{T,1}^{(1)}-X_{T,2}^{(1)}\big)\big(X_{T,1}^{(2)}-X_{T,3}^{(2)}\big)>0|X_{T,1}^{(2)},X_{T,2}^{(2)},X_{T,3}^{(2)}>t\Big) \\ &\quad -3\Pr\Big(\big(X_{T,1}^{(1)}-X_{T,2}^{(1)}\big)\big(X_{T,1}^{(2)}-X_{T,3}^{(2)}\big)<0|X_{T,1}^{(2)},X_{T,2}^{(2)},X_{T,3}^{(2)}>t\Big).\end{aligned} \tag{1.4}$$

Finally, the equivalent Pearson product-moment correlation coefficients are

$$\rho_L^{+1}(t) \; := \; \frac{cov\big(S_T, X_T^{(1)}|X_T^{(1)}>t\big)}{\sqrt{Var\big(S_T|X_T^{(1)}>t\big)Var\big(X_T^{(1)}|X_T^{(1)}>t\big)}} \tag{1.5}$$

and

$$\rho_L^{12}(t) \; := \; \frac{cov\big(X_T^{(1)}, X_T^{(2)}|X_T^{(2)}>t\big)}{\sqrt{Var\big(X_T^{(1)}|X_T^{(2)}>t\big)Var\big(X_T^{(2)}|X_T^{(2)}>t\big)}}\,. \tag{1.6}$$

If the individual claim amount has regularly varying tail, then the asymptotic behaviour of the proposed conditional measures of association is described in Theorem 1.1

Theorem 1.1. *Assume that $X \in RV_{-\alpha}$ is a positive rv. If*

a) $E(1+\epsilon)^N < \infty$ *for some* $\epsilon > 0$ *then*
   i) $\tau^{+1}(t) \sim \rho_R^{+1}(t) \sim 1$ *for all* $\alpha > 0$, *provided that* $F(\cdot)$ *is a continuous function;*
   ii) $\rho_L^{+1}(t) \sim 1$ *whenever* $\alpha > 2$*;*

b) $EN^2 < \infty$ *then*
   i) $\tau^{12}(t) \sim 1/3$ *and* $\rho_R^{12}(t) \sim 7/15$ *for all* $\alpha > 0$, *provided that* $F(\cdot)$ *is a continuous function;*
   ii) $\rho_L^{12}(t) \sim \sqrt{\dfrac{\alpha(\alpha-2)}{(5\alpha-1)(\alpha-1)}}$, *whenever* $\alpha > 2$.