Stress scenario generation for solvency and risk management

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Extended abstract:
This paper studies worst-case scenarios for probability weighted reserves in the case where we allow for mutual dependence of the interest rate and transition intensities. The dependence is indirectly introduced by the shape of the set for which we are to choose the worst combination of the interest rate and transition intensities in order to maximize the probability weighted reserve. In contrast to the vast majority of optimization problems in insurance, we do not let the worst-case (normally referred to as optimal) scenarios be adapted to a stochastic process, but instead be deterministic processes chosen within some given set. As noted in Christiansen and Steffensen (2013) worst-case scenarios can be used for calculations of surrender values, risk margins and equivalents to stress scenarios. The latter can be used for partial internal models in Solvency II.

This work extends the results of Christiansen and Steffensen (2013) to the case where we do not have the two conditions in Christiansen and Steffensen (2013, Proposition 4.1 and 4.2) about the two argmax to be constant with respect to the transition probabilities. These conditions are rather restrictive and limit the number of models to which the results of Christiansen and Steffensen (2013) apply. The calculations of the worst-case reserve in a general hierarchical disability model is for example not possible. Moreover, the results in the present paper are presented for a given starting distribution rather than for a fixed starting state. Lastly, the extension to portfolios is derived.

The main reason for this extension is to make the worst-case calculations applicable to a wide range of insurance contracts making it possible to apply the results for what is called a partial internal model in the Solvency II terminology. Like in Christiansen

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and Steffensen (2013) we solve nonlinear problems and find deterministic worst-case scenarios known at time 0 as opposed to e.g. Li and Szimayer (2014, 2011) who calculate worst-case scenarios as adapted processes (adapted to some underlying stochastic processes).

For a (real, unknown) interest rate $\phi$ and (real, unknown) transition probabilities $\mu$, we want to find a deterministic interest rate and deterministic transition probabilities $(\tilde{\phi}, \tilde{\mu})$ such that it for liabilities $L$ of a life insurance company holds that

$$P\left( L(t, \tilde{\phi}, \tilde{\mu}) \geq L(t, \phi, \mu) \right) \geq 1 - \alpha,$$

where $\alpha \in [0, 1)$. That is, we want to find a deterministic calculation basis such that the liabilities calculated with this basis with a certain probability (bigger than a certain level) lead to bigger liabilities than the ones calculated with the real (stochastic) basis. This can be obtained by choosing $(\tilde{\phi}, \tilde{\mu}) = \text{argmax}_{(\phi, \mu) \in M} L(t, \phi, \mu)$ for a set $M$ such that $P((\phi, \mu) \in M) \geq 1 - \alpha$.

As shown in Christiansen and Steffensen (2013), we can use this to obtain an upper bound for the solvency capital requirement given by

$$\sup_{(\phi, \mu) \in M} \{ L(t, \phi, \mu) \} - L(t, \phi^{BE}, \mu^{BE})$$

where $\phi^{BE}$ and $\mu^{BE}$ are best estimates for the interest rate and transition intensities, respectively.

The paper contains numerical calculations of worst-case scenarios for both individual contracts and portfolios for different types of sets (dependency structures). Furthermore, the capital requirement based on sum of individually calculated worst-case reserves, the capital requirement based on worst-cased reserves for the portfolio, and the capital requirement of the standard model of Solvency II are compared.

References


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