Cash flows and policyholder behaviour in the semi-Markov life insurance setup

Kristian Buchardt[*,†,1]
Joint work with Thomas Møller[*,†] and Kristian Bjerre Schmidt[†]

[*]Department of Mathematical Sciences, University of Copenhagen
Universitetsparken 5, DK-2100 Copenhagen Ø, Denmark

[†]PFA Pension, Sundkrogsgade 4, DK-2100 Copenhagen Ø, Denmark

March 15, 2014

Extended Abstract

Within the setup of a semi-Markov process in a finite state space, we consider a life insurance contract and the problem of valuation of prospective reserves and expected cash flows for life insurance liabilities.

First, without the modelling of policyholder behaviour, we show how to calculate the expected cash flow associated with future payments, and to that end we present a version of Kolmogorov's forward integro-differential equation, which does not seem to be well known in the semi-Markov setup.

The semi-Markov model is extended to include modelling of surrender and free policy behaviour. When we include policyholder behaviour in the model, in particular the so-called free policy option[2], an extra duration dependence arises, and the transition probabilities needed for valuation are dependent on two durations. We say that we have a setup with a double duration dependence. The main result is that we can effectively eliminate the extra duration, and we present a modified Kolmogorov forward integro-differential equation. With this result, the evaluation of the prospective reserve and the cash flow can be carried out with a computational complexity corresponding to the case without policyholder modelling, corresponding to the original semi-Markov model with

[1]Corresponding author, e-mail: buchardt@math.ku.dk

[2]"Free policy" is sometimes referred to as "paid-up policy" in the literature.

one duration. For a portfolio of life insurance contracts, this reduction, measured in time usage, is considerable.

The modelling of policyholder behaviour options, in the form of a surrender and a free policy option, are done by specifying transitions in an extended state space of the semi-Markov model, corresponding to exercises of the options. The surrender option is a right of the policyholder to cancel the contract, and receive an account value calculated on a so-called technical basis. The free policy option is a right of the policyholder to cancel future payments and let the contract continue as a policy with no premiums and with reduced benefits, where the reduction is calculated on a technical basis. An exercise of any of these options thus changes future payments. While the modelling of these options has an effect on the prospective reserve, it is of even greater importance when considering the structure of future cash flows, and the interest rate sensitivity.

We conclude by giving numerical results illustrating the cash flows for a simple life insurance policy. We show how the cash flows change due to policyholder modelling, and highlight that the dollar duration changes significantly, even though the prospective reserve might not change a lot. This is of particular interest if the cash flows are used for the hedging of interest rate risk, for example by duration-matching.

This talk is based on the paper [2]. See also [1] which contains a treatment of policyholder behaviour in the Markov setup.

References

[1] Kristian Buchardt and Thomas Møller. Duration dependence and cash flows in life insurance. (Preprint), Department of Mathematical Sciences, University of Copenhagen and PFA Pension, 2013.

[2] Kristian Buchardt, Thomas Møller, and Kristian Bjerre Schmidt. Cash flows and policyholder behaviour in the semi-markov life insurance setup. *Scandinavian Actuarial Journal*, 2014.