Personal Finance and Life Insurance under Separation of Risk Aversion and Elasticity of Substitution

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Abstract

In a classical Black-Scholes market, we establish a connection between two seemingly different approaches to continuous-time utility optimization. One approach is recursive utility optimization with Epstein-Zin preferences, studied in Duffie and Epstein (1992) and Kraft and Seifried (2010) for general preferences. The other approach is non-linear expected power utility optimization with dynamic updating, studied in this paper for an uncertain-lived investor. This approach is apt for a set-up with mortality risk and utility from inheritance, and because of the established connection for a certain-lived investor, our approach can be seen as a generalization of the recursive utility approach to a set-up with mortality risk and life insurance.

Over time, the optimal consumption and investment decisions of a certain-lived investor has been treated in various papers. An important, early example is Merton (1971) who considers time-additive utility optimization in continuous time. Richard (1975) generalized the work by Merton (1971) to include mortality risk and life insurance. Unfortunately, time-additive utility has the disadvantage that it mixes preferences for risk and preferences for inter-temporal substitution. The recursive utility approach and our approach both deal with this problem, in two seemingly different ways.

We study the optimal consumption, investment, and life insurance decision of an investor with power utility and an uncertain lifetime. To separate

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preferences for risk and preferences for inter-temporal substitution, we introduce consumption certainty equivalents, and we propose a time-global optimization problem that is about maximizing an infinite sum of infinitesimally small certainty equivalents for future consumption and inheritance. The problem is non-linear in expectation, and it is therefore time-inconsistent (see e.g. Björk and Murgoci (2010) for a description of time-inconsistency). To deal with the time-inconsistency, we search for an equilibrium control instead of a classical optimal control, and we present a verification theorem for a particular equilibrium control.

In the special case without mortality risk, we discover that our optimization approach is equivalent to recursive utility optimization with Epstein-Zin preferences. We find this interesting since our optimization problem has an intuitive interpretation as a global maximization of certainty equivalents and since recursive utility, in contrast to our approach, gives rise to severe differentiability problems. Also, our optimization approach can there be seen as a generalization of recursive utility optimization with Epstein-Zin preferences to include mortality risk and life insurance.

Keywords: Recursive utility, lifetime uncertainty, stochastic control, generalized Hamilton-Jacobi-Bellman equation.

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