## Assessing the solvency risk of insurance portfolios via a continuous time cohort model

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The assets and liabilities owned by an insurance company or pension fund are subject to various sources of uncertainty, making the assessment of their solvency risk a challenging task. Regulators – through the Solvency II Directive – are trying to steer insurance companies towards a comprehensive accounting of the risks affecting their portfolios. This increasing attention to the soundness of risk management practices, especially in the context of the Own Risk Solvency Assessment (ORSA) process, is enhancing the level of complexity of required valuation models. Proper assessment of the solvency risk of a portfolio requires indeed the modelling of many risk sources.

In this paper, we propose an ALM modelling framework which assesses the solvency risk of a run-off portfolio. Together with the well known randomness in the deaths of the policyholders in the portfolio (idiosyncratic risk), we account for the uncertainty in mortality rates themselves (systematic risk).

Given the current developing of Solvency II framework and the recent accounting rules (IASB), we require liabilities to be evaluated at fair-value. Thus, both interest-rate risk and longevity risk affect them. We couple a standard description of the interest-rate market by means of the well-known Vasicek (1977) model with a parsimonious description of mortality risk via a continuous-time cohort based stochastic model, following Luciano and Vigna (2008). This choice, in addition to being accurate in describing the evolution of mortality rates, coupled with our interest rate model, allows us to obtain the fair-value reserves of insurance liabilities in closed form. Having closed form expressions permits us to reduce computational effort and to handle

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the simulation of large portfolios with heterogeneous products per type and cohort. In our setting, an insurer can invest either in (risky) bonds and in the stock market, whose dynamics follows a geometric Brownian motion.

Our focus is on the characteristics of the funding ratio (F) at horizon T, (F(T)), which is the ratio of the value of assets (A(T)) and liabilities (L(T)), in order to assess the solvency probability of the insurance portfolio at time T,  $(\mathbb{P}[F(T) > 1])$ . In this context, we consider annuity portfolios, thus representing the typical situation of a pension fund in the de-accumulation phase, as well as portfolios which combine annuities and life insurance policies.

Our numerical simulations, calibrated to UK historical data, show that systematic longevity risk is particularly important in the long-run and needs to be hedged. We highlight that portfolio size, investment choices and solvency requirements are deeply interconnected. We finally focus on the effectiveness of natural hedging strategies. This analysis is relevant to insurers and annuity providers, as proper liability portfolio composition might be the most viable option to mitigate systematic longevity risk, given the absence of a proper market of hedging instruments. It requires adjustments both on the liabilities side and on the asset side, as its implementation alters the risk exposures to both longevity and interest-rate risk. We conclude that natural hedging techniques can effectively reduce the required solvency buffer when interest-rate risk is perfectly hedged.

Keywords: solvency, longevity, natural hedging, ALM, ORSA

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