Asymptotics for ruin probabilities in a discrete-time risk model with dependent financial and insurance risks

Yang Yang[1] and Dimitrios G. Konstantinides[2]

1. School of Math. and Stat., Nanjing Audit University, China, yyangmath@gmail.com

2. Dept. of Mathematics, University of the Aegean, Greece, konstant@aegean.gr

We consider a discrete-time insurance risk model with financial and insurance risks in a stochastic economic environment, which was proposed by Nyrhinen (2001). Suppose that the insurer invests the surplus into risk-free and risky assets, which leads to an overall positive stochastic discount factor Y_i applying over the interval $(i-1, i]$. For each positive integer n, the sum

$$S_n = \sum_{i=1}^{n} X_i \prod_{j=1}^{i} Y_j \,, \tag{0.1}$$

represents the stochastic discounted value of aggregate net losses up to time n. The infinite time ruin probability is defined by

$$\psi(x) = \lim_{n\to\infty} \psi(x,\, n) = \lim_{n\to\infty} \mathbb{P}\left(\max_{1\le k\le n} S_k > x \right) , \tag{0.2}$$

where $x \ge 0$ is interpreted as the initial wealth of the insurer.

For a distribution F, denote its upper Matuszewska index by

$$J_F^+ = -\lim_{y\to\infty} \frac{\log \overline{F}_*(y)}{\log y} \quad \text{with} \quad \overline{F}_*(y) := \liminf \frac{\overline{F}(x\,y)}{\overline{F}(x)} \quad \text{for} \quad y > 1\,. \tag{0.3}$$

Let $(X,\, Y)$ be a random vector with continuous marginal distributions F and G, then the dependence structure of X and Y is characterized in terms of a bivariate copula function $C(u,\, v)$. A function $\overline{C}(u,\, v)$, defined by the formula $\overline{C}(u,\, v) = u + v - 1 + C(1-u,\, 1-v)$, with $(u,\, v) \in [0,\, 1]^2$, is called survival copula. Assume that the copula function $C(u,\, v)$ is absolutely continuous. Denote by $C_1(u,\, v) = \frac{\partial}{\partial u} C(u,\, v)$, $C_2(u,\, v) = \frac{\partial}{\partial v} C(u,\, v)$, $C_{12}(u,\, v) = \frac{\partial^2}{\partial u\, \partial v} C(u,\, v)$. Then we define $\overline{C}_2(u,\, v) = 1 - C_2(1-u,\, 1-v)$ and $\overline{C}_{12}(u,\, v) = C_{12}(1-u,\, 1-v)$.

Assumption $\mathbf{A}_1$. *There exists a positive constant M such that*

$$\limsup_{v\uparrow 1} \limsup_{u\uparrow 1} C_{12}(u,\, v) = \limsup_{v\uparrow 1} \limsup_{u\uparrow 1} \overline{C}_{12}(1-u,\, 1-v) < M\,.$$

Assumption $\mathbf{A}_2$. *The relation* $\overline{C}_2(u,\, v) \sim u\, \overline{C}_{12}(0+,\, v)\,,\ u \downarrow 0$, *holds uniformly on* $(0,\, 1]$.

Assumption $\mathbf{A}_3$. *The relation* $C_2(u,\, v) = 1 - \overline{C}_2(1-u,\, 1-v) \to 0\,,\ u \downarrow 0$, *holds uniformly on* $[0,\, 1]$.

Denote by H the distribution of the product $X\,Y$, respectively. For each $i \ge 1$, denote by H_i the distribution of $X_i \prod_{j=1}^{i} Y_j,\ j \ge 1$.

Theorem 0.1. *In the discrete-time risk model, assume that* $\{(X_i,\, Y_i),\ i \geq 1\}$ *are i.i.d. random vectors with generic random vector* $(X,\, Y)$ *satisfying Assumptions* A_1*–*A_3*. If* $F \in \mathscr{C}$ *and* $\mathbb{E}Y^p < \infty$ *for some* $p > J_F^+$*, then, for each fixed* $n \geq 1$*, it holds that*

$$\psi(x,\, n) \sim \sum_{i=1}^{n} \overline{H}_i(x)\,. \tag{0.4}$$

Theorem 0.2. *Under the conditions of Theorem 0.1, if* $J_F^- > 0$ *and* $\mathbb{E}Y^p < 1$ *for some* $p > J_F^+$*, then it holds that*

$$\psi(x) \sim \sum_{i=1}^{\infty} \overline{H}_i(x)\,. \tag{0.5}$$

Theorem 0.3. *Under the conditions of Theorem 0.2, (0.4) holds uniformly over the integers* $\{n \geq 1\}$*.*

Define a nonnegative r.v. Y_c, which is independent of the $\{(X, Y),\ (X_i, Y_i),\ i \geq 1\}$, with the distribution G_c, given by $G_c(\mathrm{d}y) = C_1[1-,\, G(\mathrm{d}y)] = C_{12}[1-,\, G(y)]G(\mathrm{d}y)$.

Corollary 0.1. *(1) Under the conditions of Theorem 0.1, if* $F \in \mathscr{R}_{-\alpha}$ *for some* $\alpha \geq 0$*, then, for each fixed* $n \geq 1$*, it holds that*

$$\psi(x,\, n) \sim \mathbb{E}Y_c^{\alpha}\, \frac{1 - (\mathbb{E}Y^{\alpha})^n}{1 - \mathbb{E}Y^{\alpha}}\, \overline{F}(x)\,, \tag{0.6}$$

by convention, $(1 - (\mathbb{E}Y^{\alpha})^n)/(1 - \mathbb{E}Y^{\alpha}) = n$ *if* $\mathbb{E}Y^{\alpha} = 1$*.*

(2) Under the conditions of Theorem 0.2, if $F \in \mathscr{R}_{-\alpha}$ *for some* $\alpha > 0$*, then*

$$\psi(x) \sim \frac{\mathbb{E}Y_c^{\alpha}}{1 - \mathbb{E}Y^{\alpha}}\, \overline{F}(x)\,. \tag{0.7}$$

Moreover, (0.6) *holds uniformly over the integers* $\{n \geq 1\}$*.*

Keywords: consistent variation; financial and insurance risks; ruin probabilities; dependence.

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