On a Measure of Drawdown Risk*
– Extended Abstract –

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Maximum drawdown, the maximum cumulative loss from peak to trough, is one of the most widely used indicators of risk in the fund management industry, but one of the least developed in the context of probabilistic risk metrics.

A levered investor is liable to get caught in a liquidity trap: unable to secure funding after an abrupt market decline, he may be forced to sell valuable positions under unfavorable market conditions. This experience was commonplace during the 2007-2009 financial crisis and it has refocused the attention of both levered and unlevered investors on an important liquidity trap trigger, a drawdown, which is the maximum decline in portfolio value over a fixed horizon.

In the event of a large drawdown, common risk diagnostics, such as volatility, Value-at-Risk, and Expected Shortfall, at the end of the intended investment horizon are irrelevant. Indeed, within the universe of hedge funds and commodity trading advisors (CTAs), one of the most widely quoted measures of risk is maximum drawdown. However, a widely accepted mathematical methodology for forming expectations about future potential drawdowns does not seem to exist. Drawdown in the context of measures of risk as developed in Artzner et al. (1999) has failed to attract the same kind of research devoted to other more conventional risk measures.

Our purpose is to formulate a mathematically sound and practically useful measure of drawdown risk. To this end, we develop a probabilistic measure of risk capturing drawdown in the spirit of Artzner et al. (1999). Our formalization of drawdown risk is achieved by modeling the uncertain payoff along a finite path as a time-ordered random vector $X_{T_n} = (X_{t_1}, \ldots, X_{t_n})$, to which a certain real-valued functional, the *Conditional Expected Drawdown*, is applied. Technically, the random variables X_{t_i} are first transformed to the random variable $\mu(X_{T_n})$, representing the maximum drawdown within a path

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of some fixed length n. At confidence level $\alpha \in [0, 1]$, the *Conditional Expected Drawdown* CED_α is then defined to be the expected maximum drawdown given that some maximum drawdown threshold DT_α, the α-quantile of the maximum drawdown distribution, is breached:

$$\mathrm{CED}_\alpha(\mathrm{X}_{\mathrm{T_n}}) = \mathbb{E}\left(\mu(\mathrm{X}_{\mathrm{T_n}}) | \mu(\mathrm{X}_{\mathrm{T_n}}) > \mathrm{DT}_\alpha\right).$$

In the context of risk measures, CED is not a *monetary* risk metric, in the sense that it fails to satisfy the translation invariance and monotonicity axioms. It is, however, convex, which means that it promotes diversification and can be used in an optimizer. It is also homogenous of degree one, so that it supports risk attribution. Moreover, CED is a deviation measure in the sense of Rockafellar et al. (2002, 2006).

Based on these properties, drawdown risk can be integrated in the investment process in a mathematically consistent way, in terms of risk attribution and diversification analysis with respect to drawdown risk. Moreover, we show that, unlike volatility and expected shortfall, drawdown accounts for serial correlation in asset returns, and this manifests itself in the drawdown risk concentrations. Because of its convexity, CED can be optimized, and an efficient linear programming algorithm solving the CED minimization problem is derived.

Because Conditional Expected Drawdown is defined as the tail mean of a distribution of maximum drawdowns, it is analogous to Expected Shortfall, which is the tail mean of the return distribution. Hence, much of the theory surrounding Expected Shortfall carries over when moving from returns to maximum drawdowns. We will show, however, that one advantage of looking at maximum drawdown rather than return distributions lies in the fact that drawdown is inherently path dependent, whereas Expected Shortfall does not account for consecutive losses.

Having focused on theoretical foundations, we conclude by pointing out some challenges arising when moving from theory to practice, particularly when it comes to estimating and forecasting drawdown risk.

References

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