The Impact of Stochastic Volatility and Policyholder Behaviour on Guaranteed Lifetime Withdrawal Benefits

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March 15, 2014

Variable annuities (VA) are long-established investment products linked to one or a basket of several underlying reference funds or indices. They usually come with some guarantee by the policy writing institution and substantial tax advantages. In the last decade, the latest form of guarantees, termed Guaranteed Mininum Withdrawal Benefits (GMWB), has gained massive popularity. This variant allows the policyholder to withdraw the account value at periodic dates in small portions, which are guaranteed a minimum level irrespective of the underlyings' performance.
More recently, attractive features such as Guaranteed Mininum Withdrawal Benefits for Life (GLWB) and Ratchet or Roll-up riders were introduced. As for traditional insurance products, VAs commonly also include a surrender option, which can be treated to some extent as an optimal stopping problem for American options.
Since for the valuation of these contracts analytical solutions are not available, a variety of authors have studied presented methods for the former, starting eg. with Milevsky and Salisbury (2006), who considered GMWB contracts. Goals were the computation of a fair continously deducted guarantee fee, sensitivities to various capital market and other parameters and hedging possibilities and performances.
As shown by several studies (eg. Knoller et al. 2011), for a realistic model, an optimal surrender model is less adequate than some form of suboptimal withdrawal strategy of the contract holder from a hedge cost maximization point of view. The latter can result from criteria such as transaction cost, as described in Ho et al (2005), the moneyness of the guarantee (see Kling et al. (2011)) or other exogenous factors. The aim of prepayment models for mortgage-backed securities (an example is Stanton (1995)) bears similarities in this aspect.
In many institutions today, the models for the underlying asset dynamics use the assumption of stochastic volatility, which is exhibited in models such as by Hull and White (1987) or Heston (1993). Compared to the standard Black-Scholes model, these models come with substantially higher computational cost. For GLWBs, computational complexity is a major issue.

Dai, Kwok and Zong (2008) study the valuation of Guaranteed Minimum Withdrawal Benefits with a PDE approach under a stochastic control model, assuming always optimal policyholder behaviour. They show numerical results bearing a substantial sensitivity to various parameters. However, they do not consider any stochastic volatility, nor lifetime guarantee or mortality factors. As a recent alternative to employing stochastic volatility, Forsyth and Vetzal (2013) use a Markov regime switching approach. They consider the optimal surrender strategy and a transaction cost model.
In this paper, we compare the effect of stochastic volatility on the fair guarantee fee and the contract value. Also, we investigate the sensitivity of the latter to assumptions in the surrender strategy of the policyholder. An optimal surrender strategy giving an upper bound on the fair guarantee fee is compared to fair fees resulting from various surrender behaviours performing suboptimally.
In a Monte Carlo approach, the estimation of the expected continuation value requires refined methods due to the exponential problem complexity. We illustrate the application of the proven Least-Squares Monte Carlo technique as described for american options eg. in Longstaff and Schwartz (2001).
Secondary markets for life insurances have gained importance in recent years. In Germany, the introduction of an obligatory notice about their existence akin to the UK could have a notable effect, with the common clause that sale on the secondary market only be accepted after an initial contract holding period. We also study the impact of a secondary market on contract values for different assumptions on the effectiveness of the former.
We present numerical results for the contract values and fair guarantee fees using the approaches above. We conclude that the policy values are highly sensitive to various individual assumptions and parameters and should be subject to extended future research.

Keywords: Variable Annuities, Guaranteed Lifetime Withdrawal Benefits (GLWB), Valuation, Least-Squares Monte-Carlo, Optimal Stopping.

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