Tail–risk Aggregation

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Summary

Investment strategies, risk–management practices and regulation of the financial and insurance industry focus increasingly on tail–risk and relies on Value–at–Risk (VaR), and Expected Shortfall (ES) as risk measures. To assess the risk arising from a set of risk drivers, the individual risk components need to be aggregated. This requires knowledge of the dependence structure governing the risk drivers. Despite its limitations and a growing range of alternatives, the conventional Pearson correlation continues to be the most commonly adopted measure of dependence in practice.

An increasing concern in financial risk assessment is that asset price movements seem to be more closely aligned, i.e., display higher correlations, in periods of high market stress. This is supported by a various empirical studies, such as Longin and Solnik (2001), Okimoto (2008), and Mittnik (2014), and suggests that joint distributions of returns on financial assets are not normally or elliptically distributed.

Downside risk–based investment strategies and recent regulation for the insurance industry (European Commission, 2007) and the finance sector (Basel Committee on Banking Supervision, 2013) try to allow for this phenomenon by using *tail–correlation* matrices for risk aggregation. Procedures for estimating tail–correlation matrices that are associated with the particular VaR confidence level and satisfy desirable properties, such as positive semidefiniteness, have been proposed in Mittnik (2014). As correlation is a measure of linear dependence, these VaR–implied correlations can be viewed as local linear approximations of more complex dependence structures, possibly characterized by asymmetries and fat–tailedness.

Here, we generalize the approach in Mittnik (2014) and derive correlation matrices that are associated with specific regions of the support of the joint distribution governing the risk drivers. A particular focus will be on the tail

areas, in order to derive tail–correlation matrices for risk aggregation that are compatible with the ES measure. An empirical application to U.S. blue chip stocks illustrates the approach.

References

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