Pricing Currency Derivatives with Markov-modulated Lévy Dynamics

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Abstract

The aim of this research is to generalize formulas in [3] for the Esscher transform parameters using a Lévy process in the case of the log-normal distribution, which ensures that the martingale condition holds for the discounted foreign exchange rate. Until the early 1990s the existing academic literature on the pricing of foreign currency options could be divided into two categories. In the first, both domestic and foreign interest rates are assumed to be constant whereas the spot exchange rate is assumed to be stochastic. See, e.g., Jarrow et al (1981, [4]). The second class of models for pricing foreign currency options incorporated stochastic interest rates, and were based on Merton's 1973, [5]) stochastic interest rate model for pricing equity options. Unfortunately, this pricing approach did not integrate a full term structure model into the valuation framework. To our knowledge, Amin et al. (1991, [1]) were the first to start discussing and building a general framework to price contingent claims on foreign currencies under stochastic interest rates using the Heath et al. (1987) model of term structure. Mikkelsen (2001) investigated by simulation cross-currency options using market models of interest rates and deterministic volatilities for spot exchange rates. Schlogl (2002) extended market models to a cross-currency framework. Piterbarg (2005) developed a model for cross-currency derivatives such as PRDC swaps with calibration for currency options; he used neither market models nor stochastic volatility models. Takahashi et al. (2006) proposed a new approximation formula for the valuation of currency options using jump-diffusion stochastic volatility processes for spot exchange rates in a stochastic interest rates environment. Ahn et al. (2007) derived explicit formulas for European foreign exchange call and put options values when the exchange rate dynamics are governed by jump-diffusion processes. Hamilton (1988) was the first to investigate the term structure of interest rates by rational expectations econometric analysis of changes in regime. Goutte et al. (2011) studied foreign exchange rates using a modified Cox-Ingersoll-Ross model

under a Hamilton Markov regime switching framework. Zhou et al. (2012) considered an accessible implementation of interest rate models with regime-switching. Siu et al (2008) considered pricing currency options under a two-factor Markov modulated stochastic volatility model. Swishchuk et al. (2007) applied hidden Markov models for pricing options and swaps. Bo et al. (2010, [3]) discussed a Markov-modulated jump-diffusion (modeled by a compound Poisson process), for currency option pricing. We note that currency derivatives for domestic and foreign equity markets and for the exchange rate between the domestic currency and a fixed foreign currency with constant interest rates are discussed in Bjork (1998, [2]). We also mention that currency conversion for forward and swap prices with constant domestic and foreign interest rates are discussed in Benth et al. (2008).

In the research we generalize the results of [3] to a case when the dynamics of the FX rate is driven by a general Lévy process. In the conclusion, the main results of our research are as follows: 1) We generalize the formulas of [3] for Esscher transform parameters which ensure that the martingale condition for the discounted foreign exchange rate is a martingale for a general Lévy process. Using the values of these parameters we proceed to a risk-neural measure and provide new formulas for the distribution of jumps, the mean jump size, and the Poisson process intensity with respect to the measure. Pricing formulas for European call foreign exchange options are given as well (They are similar to those in [3], but the mean jump size and the Poisson process intensity with respect to the new risk-neutral measure are different); 2) We apply obtained formulas to the case of log double exponential processes; 3) We also provide numerical simulations of European call foreign exchange option prices for different parameters. Codes for Matlab functions used in numerical simulations of option prices are also provided.

Keywords : foreign exchange rate, Esscher transform, risk-neutral measure, European call option, Lévy processes, Markov Processes.

References

[1] Amin K. and Jarrow R. (1991): Pricing foreign currency options under stochastic interest rates. J. Intern. Money and Finance, 10, 310-329.

[2] Bjork T. (1998): Arbitrage Theory in Continuous Time. 2nd ed. Oxford University Press.

[3] Bo L., Wang Y. and Yang X. (2010): Markov-modulated jump-diffusion for currency option pricing. Insurance: Mathematics and Economics, 46, 461-469.

[4] Jarrow R. and Oldfield G. (1981): Forward contracts and futures contracts. J. Financial Economics, 9,373-382.

[5] Merton R. (1973): The theory of rational option pricing. Bell J. Econ. Manag. Sci., Spring, 4, 141-183.