CTE based Risk Measures and Copula Regression Using Multivariate t-Copula

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Abstract

The Conditional Tail Expectation (CTE) is arguably the most popular risk measure in quantitative risk management. It has been widely used in finance and insurance. However, most of the research on CTE focuses on a single risk variable or the case where it can be reduced to one variable. Copula regression has been carried out either under the independence structure, multivariate normality or mainly for one variable. It is of fundamental interest to address multiple risks or factors in the presence of dependence and heavy-tailedness when modeling losses.

In our research, we generalize the CTE in two aspects. The first risk measure extends the CTE to multivariate case. The second risk measure combines CTE and regression analysis.

The CTE risk measure, defined as the average amount of loss given that the loss exceeds a specified quantile, overcomes the disadvantages of Value-at-Risk (VaR) risk measure and is accepted as one of the most important risk measures in both academia and industry. In practice, it is not always the ideal case that we can reduce the risk into one dimension but instead, the targeted risk is largely affected by other factors, not just itself. Therefore, in our research, we focus on the multivariate version of the CTE as follows,

$$\mathrm{E}\left[Y|X_1 \geq t_1, ..., X_k \geq t_k\right], \tag{1}$$

where Y is the targeted risk, $X_1, ..., X_k$ represent the factors related to Y and $t_i = \mathrm{VaR}_p(X_i)$, for $i = 1, 2, ..., k$ is the Value-at-Risk of X_i at level $p \in (0, 1)$.

The essential question of regression analysis is to compute the predicted value:

$$\mathrm{E}\left[Y|X_1 = t_1, ..., X_k = t_k\right], \tag{2}$$

where Y is the response variable and $X_1, ..., X_k$ represent the covariates. The two well known regression models, the ordinary least squares and the generalized linear model, have

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restrictions on the choice of the distribution of Y; either it is a normal distribution or a member of the exponential family. In practice, the distribution of Y is not necessary to be a normal distribution and it may even have heavier tail. In our research, we express (2) as a risk measure and remove the restriction on Y. We model the dependence structure of the vector $(Y, X_1, X_2, ..., X_k)$ using a copula. This was actually first proposed in [4] but in that paper the authors only considered the case of multivariate normal (MVN) copula.

Note that in [2] and [3], the authors studied the risk measures (1) and (2) for the case $k = 1$, or the bivariate case, under different dependence structures. However, the analytical results for the case $k > 1$ are generally very difficult to obtain. In our work we study the case $k > 1$ numerically.

In order to reflect the asymptotic dependence and heavy-tailedness, we assume the losses follow multivariate t-copula (MVT) with heavy-tailed marginal distributions. The MVT copula has received increasing attention in the recent actuarial and finance literature; see for example [1]. It has been showed that the t-copula is asymptotically tail dependent, which enables us to capture the tail dependence. The computation of MVT copula is never easy because no explicit expressions are available. We numerically compute these two risk measures. We also compare the two risk measures under different dependence structures, for example, the commonly used MVN copula. As an application, we apply these two risk measures to a data set from the medical expenditure panel survey and show that the MVT copula is a better dependence model than a MVN copula.

Keywords: multivariate t-copula, conditional tail expectation, regression, risk measure, heavy-tailed, dependence structure

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