Optimal Dividends in the Dual Model under Fixed Transaction Costs

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Key words dual model; dividends; impulse control; spectrally positive Lévy processes; scale functions

1 Introduction

We solve the optimal dividend problem under fixed transaction costs in the so-called *dual model*, in which the surplus of a company is driven by a Lévy process with positive jumps (*spectrally positive Lévy* process). This is an appropriate model for a company driven by inventions or discoveries. The case without transaction costs has recently been well-studied; see [1], [2], [3], and [4]. In particular, in [5], we show the optimality of a barrier strategy (reflected Lévy process) for a general spectrally positive Lévy process of bounded or unbounded variation.

We will denote the surplus of a company by a spectrally positive Lévy process $X = \{X_t; t \geq 0\}$ whose *Laplace exponent* is given by

$$\psi(s) := \log \mathbb{E}\left[e^{-sX_1}\right] = cs + \frac{1}{2}\sigma^2 s^2 + \int_{(0,\infty)} (e^{-sz} - 1 + sz1_{\{0<z<1\}})\nu(\mathrm{d}z), \quad s \geq 0,$$

where ν is a Lévy measure with the support $(0, \infty)$ that satisfies the integrability condition $\int_{(0,\infty)}(1 \wedge z^2)\nu(\mathrm{d}z) < \infty$. We exclude the trivial case in which X is a subordinator.

Let $\mathbb{P}_x$ be the conditional probability under which $X_0 = x$ (also let $\mathbb{P} \equiv \mathbb{P}_0$), and let $\mathbb{F} := \{\mathcal{F}_t : t \geq 0\}$ be the filtration generated by X.

A (dividend) *strategy* $\pi := \{L_t^\pi; t \geq 0\}$ is given by a nondecreasing, right-continuous and $\mathbb{F}$-adapted *pure jump* process in the form $L_t^\pi = \sum_{0 \leq s \leq t} \Delta L_s^\pi$ with $\Delta L_t = L_t - L_{t-}$, $t \geq 0$. Corresponding to every strategy π, we associate a *controlled surplus* process

$$U_t^\pi := X_t - L_t^\pi, \quad t \geq 0,$$

where $U_{0-}^\pi = x$ is the initial surplus and $L_{0-}^\pi = 0$. The time of ruin is defined to be $\sigma^\pi := \inf\{t > 0 : U_t^\pi < 0\}$.

A lump-sum payment cannot be more than the available funds and hence it is required that $\Delta L_t^\pi \leq U_{t-}^\pi + \Delta X_t$ for all $t \leq \sigma^\pi$ *a.s.* Let Π be the set of all admissible strategies. The problem is to compute, for a given discount factor $q > 0$, the expected net present value (NPV) of dividends until ruin

$$v_\pi(x) := \mathbb{E}_x\Big[\int_0^{\sigma^\pi} e^{-qt}\mathrm{d}\Big(L_t^\pi - \sum_{0\le s\le t}\beta 1_{\{\Delta L_s^\pi > 0\}}\Big)\Big], \quad x \ge 0,$$

where $\beta > 0$ is the unit transaction cost, and to obtain an admissible strategy that maximizes it, if such a strategy exists. Hence the (optimal) value function is written as

$$v(x) := \sup_{\pi\in\Pi} v_\pi(x), \quad x \ge 0.$$

2 Main Results

We show that a (c_1^*, c_2^*)-policy is optimal for some $c_2^* > c_1^* \ge 0$. For $c_2 > c_1 \ge 0$, a (c_1, c_2)-policy, $\pi_{c_1,c_2} := \{L_t^{c_1,c_2}; t \ge 0\}$, brings the level of the controlled surplus process $U^{c_1,c_2} := X - L^{c_1,c_2}$ down to c_1 whenever it reaches or exceeds c_2. Define

$$v_{c_1,c_2}(x) := \mathbb{E}_x\Big[\int_0^{\sigma_{c_1,c_2}} e^{-qt}\mathrm{d}\Big(L_t^{c_1,c_2} - \sum_{0\le s\le t}\beta 1_{\{\Delta L_s^{c_1,c_2} > 0\}}\Big)\Big], \quad x \ge 0, \qquad (2.\ 1)$$

where $\sigma_{c_1,c_2} := \inf\{t > 0 : U_t^{c_1,c_2} < 0\}$ is the corresponding ruin time. The main results are given below.

Theorem 1. *There exist $0 \le c_1^* < c_2^*$ such that $v_{c_1^*,c_2^*}(x) = \sup_{\pi\in\Pi} v_\pi(x)$ for every $x \ge 0$ and the (c_1^*, c_2^*)-policy is optimal.*

In order to derive this result, we first write v_{c_1,c_2} using the *scale function*. We then show the existence of the maximizers $0 \le c_1^* < c_2^* < \infty$ that satisfy the continuous fit (resp. smooth fit) at c_2^* when the surplus process is of bounded (resp. unbounded) variation and that the derivative at c_1^* is one when $c_1^* > 0$ and is less than or equal to one when $c_1^* = 0$. These properties are used to verify the optimality of the (c_1^*, c_2^*)-policy. The levels c_1^* and c_2^* as well as the value function $v_{c_1^*,c_2^*}$ are written succinctly in terms of the scale function.

References

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