Extreme Value Analysis of the Haezendonck–Goovaerts Risk Measure with a General Young Function

Qihe Tang[1] Fan Yang[2] (Presenter)

Keywords: asymptotics; Haezendonck–Goovaerts risk measure; max-domain of attraction; (extended) regular variation; Young function

1 Aim of the research

For a risk variable X and a normalized Young function $\varphi(\cdot)$, the Haezendonck–Goovaerts (HG) risk measure for X at the confidence level $q \in (0,1)$ is defined as

$$H_q[X] = \inf_{x \in \mathbb{R}} (x + h),$$

where h solves the equation

$$\mathrm{E}\left[\varphi\left(\frac{(X-x)_+}{h}\right)\right] = 1 - q, \qquad q \in (0,1), \tag{1.1}$$

if $\Pr(X > x) > 0$ and let $h = 0$ otherwise. In the post financial crisis era, risk managers become more and more concerned with the tail area of risks due to the excessive prudence of nowadays regulatory framework. Motivated by this, we focus on the asymptotic behavior of $H_q[X]$ as $q \uparrow 1$. In Tang and Yang (2012), we implemented an asymptotic analysis for $H_q[X]$ with a power Young function for the Fréchet, Weibull and Gumbel cases. A key point of the implementation is that h can be explicitly solved for fixed x and q, which gives rise to the possibility to express $H_q[X]$ in terms of x and q. For a general Young function, however, this approach does not work any more and the problem becomes a lot harder. In the present paper, we extend the asymptotic analysis for $H_q[X]$ to the case with a general Young function and we establish a unified approach for the three extreme value cases. In doing so, we overcome several technical difficulties mainly due to the intricate relationship between the working variables x, h and q.

2 Principle Results

First we introduce several notations:

(1) L_0^{φ}: the Orlicz heart of a Young function φ.

[1]Professor, Department of Statistics and Actuarial Science, University of Iowa, Iowa City, IA, USA. E-mail: qihe-tang@uiowa.edu

[2]Assistant Professor, Actuarial Science Program, Drake University, Des Moines, IA, USA. E-mail: fan.yang@drake.edu

(2) $f(\cdot) \in \mathrm{RV}_\alpha(t_0)$: a positive measurable function $f(\cdot)$ is regularly varying at $t_0 = 0+$ or ∞ with index $\alpha \in \mathbb{R}$.

(3) $F \in \mathrm{MDA}(G_\gamma)$: a distribution function F belongs to the max-domain of attraction of the generalized extreme value distribution G_γ.

Without any additional conditions on the Young function $\varphi(\cdot)$ and the risk variable X, we can show that, the HG risk measure is equal to

$$H_q[X] = x_* + h_*, \tag{2.1}$$

where the pair (x_*, h_*) solves (1.1), and

$$\mathrm{E}\left[\varphi'\left(\frac{(X-x)_+}{h}\right)\right] = \mathrm{E}\left[\varphi'\left(\frac{(X-x)_+}{h}\right)\frac{(X-x)_+}{h}\right].$$

Note that in Bellini and Rosazza Gianin (2012), the minimizer x_* is called the Orlicz quantile of X.

The following is the main result of our work.

Theorem 2.1 *Let the Young function $\varphi(\cdot)$ be strictly convex and continuously differentiable over $\mathbb{R}_+$ with $\varphi'_+(0) = 0$, and $\varphi(\cdot) \in \mathrm{RV}_\alpha(0+) \cap \mathrm{RV}_\beta(\infty)$ for some $1 < \alpha, \beta < \infty$. Let the random variable $X \in L_0^\varphi$ and $F \in \mathrm{MDA}(G_\gamma)$ with $-\infty < \gamma < \alpha^{-1} \wedge \beta^{-1}$. In case $\gamma \leq -1$, further assume that $\varphi(\cdot)$ is twice differentiable over $\mathbb{R}_+$ such that $\mathrm{E}\left[\varphi''(\lambda Y)\right] < \infty$ for all $\lambda > 0$. Then the pair (x_*, h_*) appearing in (2.1) satisfies*

$$\overline{F}(x_*) \sim \frac{1-q}{\mathrm{E}\left[\varphi(\lambda Y)\right]} \qquad \text{and} \qquad h_* \sim \frac{a(1/\overline{F}(x_*))}{\lambda}, \qquad q \uparrow 1,$$

where $a(\cdot)$ is the auxiliary function and λ is the unique positive solution to equation

$$\mathrm{E}\left[\varphi'(\lambda Y)\right] = \mathrm{E}\left[\varphi'(\lambda Y)\,\lambda Y\right], \qquad \lambda > 0. \tag{2.2}$$

Two key steps in the proof of Theorem 2.1:

1. As $q \uparrow 1$, we have $x_* \to \hat{x}$.

2. $\lim_{q\uparrow 1} a(t_*)/h_* = \lambda$, where λ is the unique solution to equation (2.2).

References

[1] Bellini, F.; Rosazza Gianin, E. Haezendonck–Goovaerts risk measures and Orlicz quantiles. Insurance: Mathematics and Economics 51 (2012), no. 1, 107–114.

[2] Tang, Q.; Yang, F. On the Haezendonck–Goovaerts risk measure for extreme risks. Insurance: Mathematics and Economics 50 (2012), no. 1, 217–227.