Risk and Solvency of a Notional Defined Contribution public pension scheme

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Overview

Aim of this talk

The aim of this presentation is twofold:

- Show at what extent the liquidity and solvency indicators are affected by fluctuations in the financial and demographic conditions,
- Explore the issue of introducing an automatic balancing mechanism into the notional model to re-establish financial equilibrium.

Basic financing techniques

- Pay as you go (PAYG): current contributors pay current pensioners (Unfunded schemes)
- Funding: contributions are accumulated in a fund which earns a market interest rate (Funded schemes)

Benefit formulae

- Defined Benefit: Pension is calculated according to a fixed formula which usually depends on the members salary and the number of contribution years.
- Defined Contribution: Pension is dependent on the amount of money contributed and their return.

Mixing possibilities

The financing choice is present for both DB and DC pension schemes.

	Pay-as-you-go	Funding
DB	Classical social security	Classical Employee DB Plan
DC	Notional Accounts (NDCs)	Pension savings accounts

Why should we consider a pension reform?

In Belgium the following demographic changes are observed:

- Rising longevity: people are living longer and longer but retire at the same age as 50 years ago.
  - Life expectancy in 1960: 70 years
  - Life expectancy in 2011: 80 years
- Drop in fertility
  - Fertility rate in 1960: 2.58 births per woman.
  - Fertility rate in 2011: 1.84 births per woman.
- Lack of actuarial fairness: No direct link between the contributions made and amount of pension received at retirement.

Change in gross public pension expenditure over 2010-2060 (in % of GDP)

Table : Change in gross public pension expenditure over 2010-2060 (in % of GDP)

Country	2010	2020	2040	2060	Change 2010-2060
BE	11,0	13,1	16,5	16,6	5,6
DE	10,8	10,9	12,7	13,4	2,6
IT	15,3	14,5	15,6	14,4	-0,9
SW	9,6	9,6	10,2	10,2	0,6
PL	11,8	10,9	10,3	9,6	-2,2
UK	7,7	7,0	8,2	9,2	1,5
UE27	11,3	11,3	12,6	12,9	1,5

Source: European Commission - The 2012 Ageing Report

Notional Defined Contribution

- The non-financial defined contribution or notional model combines:
  - Pay-as-you-go (PAYG) financing
  - A pension formula that depends on the amount contributed and the return on it which is determined by the notional rate.
- The account is called **notional** because no pot of pension fund money exists as the system is PAYG financed.
- At retirement age: Accumulated capital $\Rightarrow$ Annuity
- The annuity takes into account:
  - Life expectancy of the individual
  - The indexation of pensions
  - The technical interest rate

Main Advantages and Shortcomings of the NDC

Main Advantages

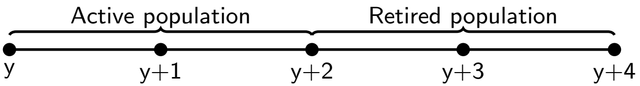
- Portability of pension rights between jobs, occupations and sectors is permitted.
- Level of benefits is known at all moments and allows to take decisions more wisely.
- It promises to deal with the effects of population ageing more or less automatically.
- Arbitrariness in benefit indexation rules and adjustment factors is avoided.

Shortcomings

- The problem of demographic change is not fully dealt with.
- In a scenario with a fixed contribution rate and a persistent rise in longevity, the size of the pension tends to decrease.
- If the notional rate is less than the market return the individual might consider the existence of an implicit cost (tax) equal to the difference in return.
- It is not solvent or liquid in general.

The Model

Age: $x = y, y + 1, y + 2, y + 3, y + 4$



The highest age to which it is possible to survive is $y + 4$.
The choice of four generations is not arbitrary:

- Introduces heterogeneity in the contributions: two generations with different demographic histories coexist.
- Introduces heterogeneity in the expenditure: mortality and indexation issues are considered.

The Model

Population at time t:
$l(x,t) = l(y, t-x+y)p(x,t) = l(y,0)\exp\left(\sum_{i=1}^{t-x+y} R_i\right)p(x,t)$
where:

- $l(y, t-x+y)$=Entry population at time $t-x+y$
- $p(x,t)$=Time-dependent survival probability to attain age x at time t

Wages at time t: $S(x,t) = S(x,0)\exp\sum_{i=1}^{t}\gamma_i$

$\hookrightarrow$ The following stochastic processes are defined in the probability space $(\Omega, \mathcal{F}, P)$:
R_i =increase rate in the entrant population during period $i-1$ to i
γ_i = increase rate of the salaries during period $i-1$ to i

$\Rightarrow$ Further assumption: no mortality risk until retirement. Thus:

- $p(x,t) = 1$ for $x = y, y+1, y+2$
- $p(y+3, t) = p_t$ for simplicity.

Contributions and notional rate

At time t, all members of the active population contribute a rate π of their salaries to the pension system:

$$C(t) = \pi S(y,t)l(y,t) + \pi S(y+1,t)l(y+1,t)$$
$$= \pi l(y,0)\exp\left(\sum_{i=1}^{t}\gamma_i + \sum_{i=1}^{t-1} R_i\right)K_C(t)$$
$$where: \;\; K_C(t) = S(y,0)e^{R_t} + S(y+1,0)$$

The notional factor $I(t)$ is taken as the changes in the total contribution base

$$I(t) = \frac{C(t)}{C(t-1)} = e^{\gamma_t + R_{t-1}}\frac{K_C(t)}{K_C(t-1)}$$

This notional rate is affected by both salary and demographic risks.

Pension calculation and expenditure

The sum of all individual contributions are indexed at the notional rate. Its accumulated value at retirement age corresponds to the notional capital $NDC_{CO}(y+2,t)$.
The initial pension is based on this notional capital and the annuity a_t at time of retirement t:

$$P(y+2,t) = \frac{NDC_{CO}(y+2,t)}{a_t l(y+2,t)}$$

The expenditure on pension becomes:

$$\begin{aligned} O(t) &= P(y+2,t)l(y+2,t) + P(y+2,t-1)\Lambda^*(t)l(y+3,t) \\ &= C(t)K_O(t) \end{aligned}$$

The indexation rate $\Lambda^*(t)$ ensures actuarial **fairness** in each cohort.
The expenditure on pension $O(t)$ at time t is thus proportional to the contributions made at the same period.

Liquidity and Solvency indicators

- Most natural way to study the liquidity is to compare income and expenses, i.e., $LR_t = \frac{C(t)+F^-(t)}{O(t)}$; where $F^-(t)$=is a buffer fund.
- As previously seen expenses are proportional to the income under this framework.
- Even if longitudinal equilibrium may be attained, cross-sectional equilibrium is not guaranteed.

Result 1

Contributions are in general not equal to the expenditure on pensions in the presented 4-period OLG unfunded dynamic model, i.e., $C(t) \neq O(t)\ \forall t$.

$\rightarrow$ Equality is only found when the population is in steady state.
$\rightarrow$ The population in Europe is not in steady state but it is rather dynamic.

Liquidity and Solvency indicators

Another way of assessing the health of the pension system is through the solvency ratio, based on the swedish system:

$$SR_t = \frac{Assets + F^{-}(t)}{V(t)}$$

where:

- $F^{-}(t)$=is a buffer fund.
- $V(t) = \sum_{x=y}^{y+3} NDC(x,t) = C(t)K_V(t)$

where: $NDC(x,t)$ is the accumulated notional capital for all ages.
$\rightarrow$ **Problem**: 1^{st} pillar pensions are mostly unfunded
$\rightarrow$ How can we estimate this non-existent asset?
$\Longrightarrow$ The assets are estimated according to some accounting measure called the Contribution Asset.

The Contribution Asset

- The valuation of the Contribution Asset has been derived for the case of a steady state scenario.
- Also used in practice, where reality hardly follows the stationary assumptions.
- This does not mean that the contribution asset remains constant over time, as these changes are included once they happen.
- It is inaccurate, but a useful tool.
- Calculated as the product of the current contribution base times the turnover duration.

The Swedish Solution

The Contribution Asset is thus:

$$CA(t) = C(t)TD(t) = C(t)(A_t^R - A_t^C)$$

$$A_t^R = \frac{\sum_{x=y+2}^{y+3} xP(x,t)l(x,t)}{\sum_{x=y+2}^{y+3} P(x,t)l(x,t)} = \text{weighted average age for the pensioners}$$

$$A_t^C = \frac{\sum_{x=y}^{y+1} xC(x,t)l(x,t)}{\sum_{x=y}^{y+1} C(x,t)l(x,t)} = \text{weighted average age for the contributors}$$

Same problem as before, this accounting measure only gives equilibrium if the population is in steady state:

Result 2

Contribution asset is in general not equal to the liabilities in the presented 4-period OLG unfunded dynamic model, i.e., $CA(t) \neq V(t)\ \forall t$

The purpose of an ABM

Its purpose is to provide 'automatic financial stability' in the sense that it should adapt to shocks without legislative intervention.
Some questions arise:

- What type of ABM should be applied?
- Will retirees and contributors be affected in the same way?
- Should ABM mechanism be symmetric or asymmetric?

Symmetric → affects under both and good economic scenarios.
Assymetric → affects only in bad times allowing for surpluses to accumulate.

Introduction of an Automatic Balance Mechanism

- As seen in the previous sections, both liquidity and solvency are not guaranteed by the NDC framework;
- An Automatic Balance Mechanism (ABM) $B_{LR}(t)$ is thus introduced through the notional rate: $I_x(t) = I(t)B_x(t)$ for x=LR,SR.
- For the liquidity case is: $B_{LR}(s) = \frac{C(s)+F^{-}(t)}{C(s)K_O^{LR}(s)}$
- For the solvency case is: $B_{SR}(s) = \frac{CA(s)+F^{-}(s)}{V(s)}$

$\rightarrow$ **Issue?**: How can we choose between these two ABM?
$\rightarrow$ We aim to choose the ABM which has a lower variance.
The ABMs will first be applied at time t.

Definition of the processes

The demographic and salary processes follow a geometric Brownian motion:

$$D_t = \frac{l(y,t)}{l(y,t-1)} = e^{R_t} = e^{R - \frac{\sigma_R^2}{2} + \sigma_R (w_R(t) - w_R(t-1))}$$

$$S_t = \frac{S(x,t)}{S(x,t-1)} = e^{\gamma_t} = e^{\gamma - \frac{\sigma_\gamma^2}{2} + \sigma_\gamma (w_\gamma(t) - w_\gamma(t-1))}$$

with:

- $\mathrm{E}[w_R(s) w_\gamma(s)] = \rho s$
- $\mathrm{E}[w_x(j) - w_x(k)] = 0$ for $x = \gamma, R$ for $j \neq k$
- $\mathrm{E}[(w_R(j) - w_R(k))(w_\gamma(j) - w_\gamma(k))] = 0$ for $j \neq k$
- $Cov(S_s, D_s) = e^{R+\gamma+\frac{\sigma_R^2+\sigma_\gamma^2}{2}} \left(e^{\rho\sigma_R\sigma_\gamma} - 1\right)$
- $Cov(D_j, D_k) = 0$ for $j \neq k$
- $Cov(D_j, S_k) = 0$ for $j \neq k$

The notional rate becomes:

$$I(s) = \frac{C(s)}{C(s-1)} = S_s D_{s-1} \frac{S(y,0)D_s + S(y+1,0)}{S(y,0)D_{s-1} + S(y+1,0)}$$

Joint distribution

The joint distribution of a random vector $X = (D_{t-3}, ..., S_s, D_s)$ is thus:

$$f_X(x) = \prod_{j=t-3}^{t} f_{D_i}(d_i) \prod_{j=t+1}^{s} f_{S_j,D_j}(s_j, d_j) \text{for } s \geq t+1$$

$$\text{where } S_s D_s \sim logN(R + \gamma - \frac{\sigma_R^2 + \sigma_\gamma^2}{2}, \sigma_{R,\gamma}^2)$$

$$\text{with } \sigma_{R,\gamma}^2 = \sigma_R^2 + \sigma_\gamma^2 + 2\rho\sigma_R\sigma_\gamma$$

The joint density function of (S_s, D_s) is:

$$f_{S_s,D_s}(x,y) = \frac{1}{xy\sqrt{|\Sigma|}} e^{-\frac{1}{2|\Sigma|}\left((\log z - \mu)' \Sigma^{-1} (\log z - \mu)\right)} \text{ for } xy > 0$$

$$\text{with: } \log z = \begin{pmatrix} \log x \\ \log y \end{pmatrix}, \ \mu = \begin{pmatrix} R - \frac{\sigma_R^2}{2} \\ \gamma - \frac{\sigma_\gamma^2}{2} \end{pmatrix} \Sigma = \begin{pmatrix} \sigma_\gamma & \rho\sigma_\gamma\sigma_R \\ \rho\sigma_\gamma\sigma_R & \sigma_R \end{pmatrix}$$

$$|\Sigma| = \text{determinant of variance-covariance matrix } \Sigma$$

Calculation of the variance for the ABM_{LR}

The expected value of the k^{th} power of the liquidity-ratio based ABM is thus:

$$\begin{aligned}\mathrm{E}[B_{LR}(s)^k] &= \mathrm{E}[g_{LR}(t,s,x_1,...,x_n)^k] \\ &= \int_0^{\infty} ... \int_0^{\infty} g_{LR}(t,s,x_1,...,x_n)^k f_X(x_1,...,x_n)dx_1...dx_n\end{aligned}$$

where :

$$g_{LR}(t,s,x) = \frac{1+f(t,s)}{K_O^{LR}(s)} \text{ if symmetric}$$

$$g_{LR}(t,s,x) = \mathrm{Min}\left[\frac{1+f(t,s)}{K_O^{LR}(s)}, 1\right] \text{ if asymmetric}$$

The function g is reduced to $g_{LR}(s) = \frac{1}{K_O^{LR}(s)}$ if the fund is equal to 0 when the ABM is first applied and if the ABM is symmetric.

Calculation of the variance for the ABM_{SR}

The expected value of the k^{th} power of the solvency-ratio based ABM is thus:

$$\mathrm{E}[B_{SR}(s)^k] = \mathrm{E}[g_{SR}(t,s)^k] = \int_0^{\infty} ... \int_0^{\infty} g_{SR}(t,s,x_1,...,x_n)^k f_X(x_1,...,x_n) dx_1 ... dx_n$$

where :

$$g_{SR}(t,s,x) = \frac{TD(s) + f(t,s)}{K_V^{SR}(s)} \text{ if symmetric}$$

$$g_{SR}(t,s,x) = \text{Min}\left[\frac{TD(s) + f(t,s)}{K_V^{SR}(s)}, 1\right] \text{ if asymmetric}$$

Numerical illustration

The variances and expected values will be studied in 3 different scenarios for both ABM:

1. Base: No longevity trend, $p_t = p \ \forall t$;
2. Up: Upward longevity trend, $p_t > p_{t-1} \ \forall t$;
3. Down: Downward longevity trend, $p_t < p_{t-1} \ \forall t$.

Furthermore, the impact of the exogenous shock δ will be studied for the three scenarios for both ABM by setting $D_t^* = D_t e^{\delta}$.
The following assumptions are taken:

$$\begin{array}{ll} R = 0.25\% & \gamma = 1.5\% \\ \sigma_R = 5\% & \sigma_\gamma = 10\% \\ S(y,0) = 30,000 & S(y+1,0) = 45,000 \\ \rho = -0.25 & p_0 = 0.5 \\ i = 2\% & \delta = 5\% \end{array}$$

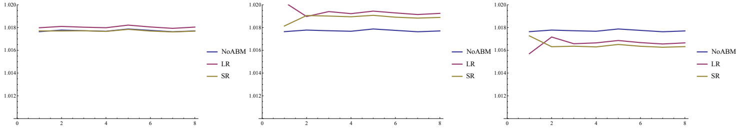
Finally, two cases will be studied:

- Case 1: prospective mortality is used and the system is **fair**
- Case 2: current mortality is used and indexation doesn't adapt to the observed longevity experience $\rightarrow$ system is **not fair**

The variance of the **fund** will also be studied.

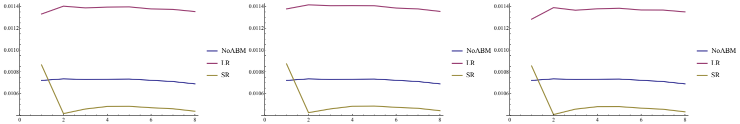
Numerical illustration-No baby boom-Case 1

Figure : Expected Value of the Notional factor with ABM - No baby boom - Symmetric



(a) Base scenario (b) Up scenario (c) Down scenario

Figure : Variances of the Notional factor with ABM - No baby boom - Symmetric



(a) Base scenario (b) Up scenario (c) Down scenario

Numerical illustration-Variances of the Notional Factor

Table : Sum of the variances of the Notional Factor - No Baby Boom

SYMMETRIC							
Case 1				Case 2			
	Base	Up	Down		Base	Up	Down
No ABM	0,08578	0,08578	0,08578	No ABM	0,08578	0,08578	0,08578
LR	0,09102	0,09113	0,09089	LR	0,09102	0,09078	0,09126
SR	**0,08408**	**0,08411**	**0,08404**	SR	**0,08408**	**0,08404**	**0,08410**
NO SYMMETRIC							
Case 1				Case 2			
	Base	Up	Down		Base	Up	Down
No ABM	0,08578	0,08578	0,08578	No ABM	0,08578	0,08578	0,08578
LR	0,08630	0,08609	0,08661	LR	0,08630	0,08644	0,08619
SR	**0,08529**	**0,08545**	**0,08509**	SR	**0,08529**	**0,08521**	**0,08536**

The same conclusions hold for the Baby Boom case.

Numerical illustration-Variances of the Fund

Table : Variance of the Fund - No Baby Boom

SYMMETRIC							
Case 1				Case 2			
	Base	Up	Down		Base	Up	Down
No ABM	0,00102	0,00101	0,00106	No ABM	0,00102	0,00103	0,00102
LR	**0,00000**	**0,00000**	**0,00000**	LR	**0,00000**	**0,00000**	**0,00000**
SR	0,00056	0,00057	0,00056	BR	0,00056	0,00056	0,00056
NO SYMMETRIC							
Case 1				Case 2			
	Base	Up	Down		Base	Up	Down
No ABM	0,00102	0,00101	0,00106	No ABM	0,00102	0,00103	0,00102
LR	**0,00070**	0,00083	**0,00058**	LR	**0,00070**	**0,00063**	0,00078
SR	0,00073	**0,00080**	0,00066	SR	0,00073	0,00068	**0,00077**

In the Baby Boom case the choice of a no symmetric ABM is not straightforward. It highly depends on the studied scenario.

Interpretation of the results

- The Solvency Ratio ABM reduces the variances of the **notional factor** in all scenarios and cases. Furthermore, the following relation is observed: $\sum_{j=t}^{t+7} Var[I_{SR}(j)] < \sum_{j=t}^{t+7} Var[I(j)] < \sum_{j=t}^{t+7} Var[I_{LR}(j)]$.
- The introduction of an ABM, both LR and SR, reduces variance of the **fund**.
- The Liquidity Ratio ABM sets the variance of the **fund** to 0 when symmetric. The following relation is observed: $\sum_{j=t}^{t+7} Var[f_{LR}(j)] < \sum_{j=t}^{t+7} Var[f_{SR}(j)] < \sum_{j=t}^{t+7} Var[f(j)]$.
- The Solvency Ratio ABM reduces the variance of the **fund** when asymetric in the No baby boom scenario. In this case it holds that, $\sum_{j=t}^{t+7} Var[f_{SR}(j)] < \sum_{j=t}^{t+7} Var[f_{LR}(j)] < \sum_{j=t}^{t+7} Var[f(j)]$.
- The choice of an asymmetric ABM under a Baby boom scenario is not straightforward. It highly depends on the studied scenario.

Further research

- Annuity design:
  - Choice between different levels of indexation;
  - Choice between projected or observed mortality values;
  - Choice of adjustments, if any, according to the real mortality experience.
- Influence of the decisions if a **mixed** plan: optimal choice between funding and PAYG.
- Pension reform **transition**: Cost of this transition and ways of optimizing it.
- NDC plans with **minimum pensions**: Calculation of the cost of the guarantee through option pricing.

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Thank you!