Dynamic Hybrid Products in Life Insurance: Assessing the Policyholders’ Viewpoint

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Introduction: Motivation

• Currently: Low interest rates, volatile capital markets, regulation, cost pressure, demographic development, changing consumer needs,…

➢ Innovations in life & pension of high relevance

➢ Dynamic hybrid products:

  – Ensure guarantee by a periodical rebalancing process (CPPI) of account value between 1) conventional reserves (“risk-free” investment), 2) guarantee fund (at most 20% loss per period, 3) equity fund (risky)

  – Combine stability of traditional life insurance (conventional policy reserves) with upside potential of unit-linked policies (through investing in a guarantee and / or equity fund)
Introduction: Aim of paper

• But: To what extent do interaction effects arise between traditional policies and dynamic hybrids?
• What is the impact of these interactions on fair value, risk, and policyholders’ willingness-to-pay?

➢ Provide a model for a life insurer selling
  - Traditional participating life insurance contracts (PLI) and
  - Dynamic hybrid products (DHP)

➢ Examine the impact of dynamic hybrid products on
  - Fair value and insurer’s risk situation
  - Policyholders’ willingness to pay using mean-variance preferences

➢ Study interaction effects between PLIs and DHPs
Model framework: Insurance company

- Consider life insurer with a product portfolio comprising
  - **PLIs**: Invest entirely in the actuarial policy reserves (**PR**)
  - **DHPs**: Contracts’ funds are dynamically allocated between reserves, a guarantee fund (**GF**) and an equity fund (**EF**)

- Simplified balance sheet at time \( t \):

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_{\text{long-term}} )</td>
<td>( PR^{PLI} )</td>
</tr>
<tr>
<td>( A_{\text{short-term}} )</td>
<td>( PR^{DHP} )</td>
</tr>
<tr>
<td>( GF^A )</td>
<td>( GF^L )</td>
</tr>
<tr>
<td>( EF^A )</td>
<td>( EF^L )</td>
</tr>
<tr>
<td>Buffer</td>
<td></td>
</tr>
</tbody>
</table>

Policy reserves annually compounded at least with *guaranteed interest rate* (and surplus generated by insurer’s investment portfolio)

- Total account value (**AV**) of hybrids
  - Dynamic (short-term, procyclical) reallocation!
    - Policy reserves (**PR^{DHP}**)
    - Guarantee fund (**GF**)
    - Equity fund (**EF**)
Model framework: Development of assets and PLI

- Development of policy reserves \( PR_{(t+1)^-}^{PLI/DHP} = PR_{t^+}^{PLI/DHP} \cdot (1 + r_t^P) \)
  - with \( r_t^P = \max \left( r^G, \alpha \cdot \left( \frac{B_t}{PR_{t^+}^{PLI} + PR_{t^+}^{DHP}} - \gamma \right) \right) \)
  - \( B_{t^+} \): buffer = company’s total assets \( (A_t) \) minus policyholders’ accounts
    \[
    B_{(t+\Delta t)^-} = A_{(t+\Delta t)^-} - PR_{(t+\Delta t)^-}^{PLI} - PR_{(t+\Delta t)^-}^{DHP} - GF_{(t+\Delta t)^-}^{L} - EF_{(t+\Delta t)^-}^{L}
    \]

- Investments all evolve according to a geometric Brownian motion
  \[
  dl_i^l = \mu_i \cdot l_i^l \cdot dt + \sigma_i \cdot l_i^l \cdot dW_{t,i}^P, \quad i = 1,2,3
  \]
- Guarantee fund equivalent to equity fund with a hedge that ensures a maximum loss of \( \lambda \) percent within one period \( (\Delta t) \)
Model framework: Dynamic hybrid products

- DHP guarantee to policyholders (single premium $P$): $G_T^{DHP} = x \cdot P^{DHP}$
- Distribution of account value (Kochanski and Karnarski, 2011):

$$
PR_{t^r}^{DHP} = \begin{cases} 
G_{t+\Delta t}^{DHP} \cdot (1 - \lambda) \cdot AV_{t^r}^{DHP} / (1 + r^G)^{\Delta t} - 1 + \lambda, & \text{if } G_{t+\Delta t}^{DHP} > 1 \\
0, & \text{otherwise}
\end{cases}
$$

$$
GF_{t^r}^{L} = \begin{cases} 
AV_{t^r}^{DHP} - PR_{t^r}^{DHP}, & \text{if } G_{t+\Delta t}^{DHP} > 1 \\
G_{t+\Delta t}^{DHP} / (1 - \lambda), & \text{otherwise}
\end{cases}
$$

$$
EF_{t^r}^{L} = AV_{t^r}^{DHP} - PR_{t^r}^{DHP} - GF_{t^r}^{L}
$$

- Concrete shifting mechanism depends on account value $G_{t+\Delta t}^{DHP}$ needed to ensure that guarantee can be met at maturity (=> product design!)
Model framework: Dynamic hybrid products (cont.)

- Two types of shifting mechanisms:
  - **Less risky** (constant guarantee with higher portion of “riskless assets” in the portfolio, i.e. policy reserves and guarantee fund):
    \[
    G_{t+\Delta t}^{DHP} = G_T^{DHP} = x \cdot P_T^{DHP} \quad \forall t \in \{0, \Delta t, \ldots, T\}
    \]
  - **More risky** (discounted maturity guarantee with higher portion in the “risky asset”, i.e. fund investments):
    \[
    G_{t+\Delta t}^{DHP} = G_T^{DHP} \cdot \left(1 + r^G\right)^{-(T-t-\Delta t)} \quad \text{with} \quad G_T^{DHP} = x \cdot P_T^{DHP}
    \]

- Different product designs in the market; strong impact on risk-return profiles
Model framework: Development of liabilities

- Buffer at the end of a the period $t$ given by

$$B_{(t+\Delta t)^-} = A_{(t+\Delta t)^-} - PR_{PLI}^{(t+\Delta t)^-} - PR_{DHP}^{(t+\Delta t)^-} - GF^L_{(t+\Delta t)^-} - EF^L_{(t+\Delta t)^-}$$

- At maturity $T$, buffer is distributed to equityholders and policyholders
  - **Equityholders** receive a buffer payback
    $$BP_{T^-} = \max\left(\min\left(B_{T^-}, B_0 \cdot (1+b)\right), 0\right)$$
  - **Policyholders** receive remainder as an **optional terminal bonus**
    $$TB_{T^-} = \max\left(0, B_{T^-} - BP_{T^-}\right)$$
  - **Terminal bonus** is distributed between PLIs and DHPs ($i = PLI, DHP$)
    $$TB^i_{T^-} = \left(\frac{T/\Delta t}{\sum_{k=1}^{T/\Delta t} PR^i_{(k\cdot\Delta t)^-}} \right) \left(\frac{T/\Delta t}{\sum_{k=1}^{T/\Delta t} PR_{PLI}^{(k\cdot\Delta t)^-} + PR_{DHP}^{(k\cdot\Delta t)^-}}\right)$$
Model framework: Fair valuation and risk measurement

• Buffer interest rate $b$ is calibrated to ensure a fair situation for equityholders

$$B_0^+ = E^Q \left( B_{P^T} \cdot e^{-T \cdot r_t} \right)$$

• Shortfall probability

$$SP = P \left( T_s \leq T \right), \quad T_s = \inf \left\{ t : A_{t-}^{long-term} + A_{t-}^{short-term} < PR_{t-} \right\}, \; t = 1, \ldots, T$$

• Total payout to PLI policyholders

$$V_{T}^{PLI} = \left( PR_{T-}^{PLI} + TB_{T-}^{PLI} \right) \cdot \mathbf{1}\{T_s > T\} + RF_{t-}^{PLI} \cdot \mathbf{1}\{T_s = t\}$$

• Total payout to DHP policyholders

$$V_{T}^{DHP} = \left( AV_{T-}^{DHP} + TB_{T-}^{DHP} \right) \cdot \mathbf{1}\{T_s > T\} + RF_{t-}^{DHP} \cdot \mathbf{1}\{T_s = t\}$$
Model framework: The policyholder’s perspective

• Present values from the policyholders’ perspective

\[ PV_0^{PLI} = E^Q \left( \left( PR_T^{PLI} + TB_T^{PLI} \right) \cdot e^{-T \cdot r} \cdot 1\{T_s > T\} \right) + E^Q \left( RF_t^{PLI} \cdot e^{-t \cdot r} \cdot 1\{T_s = t\} \right) \]

\[ PV_0^{DHP} = E^Q \left( \left( AV_T^{DHP} + TB_T^{DHP} \right) \cdot e^{-T \cdot r} \cdot 1\{T_s > T\} \right) + E^Q \left( RF_T^{DHP} \cdot e^{-t \cdot r} \cdot 1\{T_s = t\} \right) \]

• Maximum willingness to pay (mean-variance preferences)

\[ WTP_0^{\Phi,j} = e^{-r \cdot T} \cdot \left( E(V_T^j) - \frac{a}{2} \cdot \sigma^2(V_T^j) \right), \ j = PLI, DHP \]
### Numerical results: Input parameters

- **Input parameters:**

<table>
<thead>
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<th>Single premiums</th>
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<td>Participating life insurance</td>
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<td>Dynamic hybrid products</td>
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| Guaranteed interest rate               | 1.75% |
| Length of a period $\Delta t$          | one month |
| Contract duration in years             | 10    |
| Maximal loss of the guarantee fund per period | 0.2   |
Numerical results: Case without dynamic hybrids

Value of traditional contracts = premium of 100 (=> calibrated to get fair contracts)

Portion of dynamic hybrid products in the portfolio is zero, single premium of trad. contracts = 100

Insurer's shortfall probability

Varying guaranteed interest rates (for funds in policy reserves)
Numerical results: Case with dynamic hybrids

Now introduce dynamic hybrids, equal portions, i.e. single premium = 100 for both contracts

⇒ Policies are no longer fair
⇒ Value and risk level strongly depend on guarantees (dynamic hybrid guarantee level and guaranteed interest rate for conventional policy reserves)

Value of traditional contracts

Value of dynamic hybrids

Dynamic hybrid guarantee level: money-back guarantee

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Numerical results: Impact of guarantees

Now introduce dynamic hybrids, equal portions, i.e. single premium = 100 for both contracts

\[ P^{DHP} = 100, \quad G^{DHP} = 0.5 \cdot P^{DHP} \]

value and risk level strongly depend on guarantees (dynamic hybrid guarantee level and guaranteed interest rate for conventional policy reserves)

⇒ Policies are no longer fair
⇒ Value of traditional contracts
⇒ Value of dynamic hybrids

Dynamic hybrid guarantee level: 50% of upfront premium
Numerical results: Portfolio composition

But: concrete effects depend on the portfolio composition!

Value of trad. contracts => fair

Shortfall probability given fair contracts:
⇒ Minimum for approx. 125:75-portion in the portfolio (for considered example)

Dynamic hybrid: money-back guarantee

Value of dynamic hybrid contracts => fair

Keep total premium volume fix: \( P_{\text{PLI}} = 200 - P_{\text{DHP}} \)
Numerical results: Willingness to pay for **PLIs** and **DHPs**

- Less risk averse policyholder (full money-back guarantee)

![](image)

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Numerical results: Willingness to pay for PLIs and DHPs

- More risk averse policyholder (full money-back guarantee)

\[ \text{WTP}^{\text{PLI}} = \text{WTP}^{\text{DHP}} \]

\[ a=0.01, \ G_t^{\text{DHP}} = 1.0 \cdot P^{\text{DHP}} \text{ (less risky shifting)} \]

\[ a=0.01, \ G_t^{\text{DHP}} = 1.0 \cdot P^{\text{DHP}} \text{ (more risky shifting)} \]
Numerical results: Willingness to pay for PLIs and DHPs

- The impact of portfolio composition on the policyholders' WTP

![Graphs showing the willingness to pay for PLIs and DHPs with different levels of risk aversion.](image)
Numerical results: Willingness to pay for PLIs and DHPs

- The impact of portfolio composition on the policyholders’ WTP
Summary

• Model and assess portfolio effects for innovative dynamic hybrids

• Results emphasize strong interaction effects with traditional policies
  ➢ Consideration of the portfolio as a whole is vital when introducing new products
  ➢ Higher guarantee (i.e. riskiness of shifting mechanism, DHP guarantee level, guaranteed interest) does not necessarily imply an increase in consumers’ willingness to pay or present value

• DHPs:
  ➢ Minimum guarantee must be offered for \( WTP^{DHP} > P^{DHP} \), otherwise contracts are not purchased
  ➢ Contracts’ attractiveness strongly depends on contract design

• PLIs:
  ➢ \( WTP \) remained almost unchanged; traditional contract payoffs low volatile and stable
Thank you very much for your attention!
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Numerical results: Impact of surplus participation rate

\[ P^{DHP} = 100, \ G^{DHP} = 1.0 \cdot P^{DHP} \]

\[ \alpha \]

present value

shortfall probability (right axis)

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Numerical results: Average partition of the account value

\[ G^{DHP} = 1.0 \cdot P^{DHP}, \quad r^G = 0.01 \]

\[ G^{DHP} = 1.0 \cdot P^{DHP}, \quad r^G = 0.025 \]
Numerical results: Average partition of the account value

\[ G^{DHP} = 0.75 \cdot P^{DHP}, \quad r^G = 0.01 \]

\[ G^{DHP} = 0.5 \cdot P^{DHP}, \quad r^G = 0.01 \]
Numerical results: Average partition of the account value

\[ G_{DHP}^{DHP} = 1.0 \cdot P_{DHP}^{DHP}, \alpha = 0.1 \]

\[ G_{DHP}^{DHP} = 1.0 \cdot P_{DHP}^{DHP}, \alpha = 0.7 \]
Numerical results: Willingness to pay for PLIs and DHPs

- More risk averse policyholder for a partial money-back guarantee