The Effect of Risk Preferences on Equity-Linked Life Insurance with Surrender Guarantees

Christian Hilpert

Bonn University

8th Actuarial Science and Finance Conference
Samos, 29th May - 1st June 2014

Main Result

Loss-averse preferences hugely influence the surrender behavior compared to:

- Optimal surrender behavior/optimal stopping
- Standard expected utility
- Surrender behavior implied by Cox processes.

Conclusion

Behavioral finance provides evidence that people are boundedly rational and don't make monetarily optimal choices.

Previous insurance literature argues that policyholders either **surrender optimally**, or treat boundedly rationality through **exogenous stochastic processes**.

This paper prices equity-linked life insurance under loss-averse preferences and derives the surrender behavior of policyholders.

Agenda

Finance and Insurance Markets

Policyholder's Preferences

Results

Financial and Insurance Market

- **Financial Market**:

$$dS_t = \mu S_t dt + \sigma S_t dW_t^{\mathbb{P}}, \qquad S_0 > 0,$$
$$dB_t = r B_t dt, \qquad B_0 = 1.$$

- **Survival benefit:**

$$\Phi(S_T) = \alpha P \cdot \max\left\{(1+g)^T, \left(\frac{S_T}{S_0}\right)^k\right\}.$$

- **Surrender benefit:**

$$L(t) = \alpha P \cdot (1+h)^t \cdot (1-\beta_t).$$

Agenda

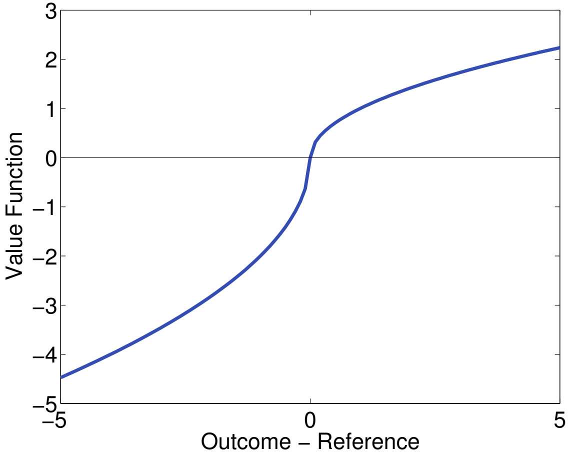
Finance and Insurance Markets

Policyholder’s Preferences

Results

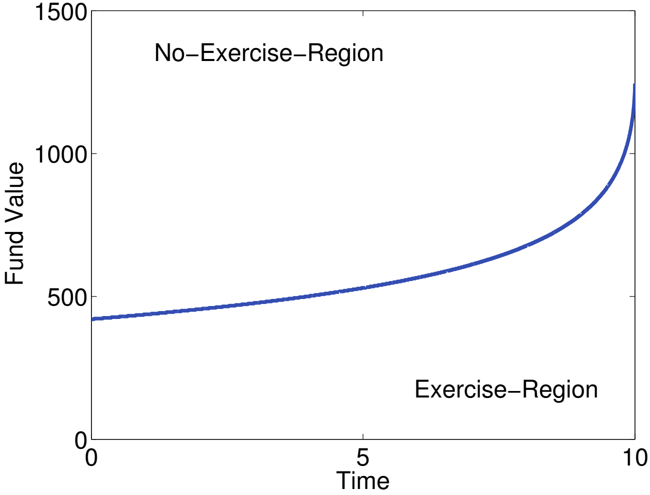
Utility function

$$U(x) = \begin{cases} (x-R)^{\gamma}, & x \geq R \\ -\lambda\left(-(x-R)\right)^{\gamma}, & x < R \end{cases}$$



How should the policyholder surrender optimally?

Rational Exercise



How does the policyholder surrender under risk preferences?

Computation of Utility

The policyholder maximizes his expected utility:

$$J\left(t, S_t, \tau^B\right) = \mathbb{E}_{t,S_t}\left[U\left(e^{r(T-\tau^B)}\Psi\left(\tau^B, S_{\tau^B}\right)\right)\right].$$

with

$$\Psi\left(\tau^B, S_{\tau^B}\right) = \begin{cases} L\left(\tau^B\right) & \tau^B < T \\ \max\left\{\Phi\left(S_T\right), L(T)\right\} = \Phi\left(S_T\right), & \tau^B = T. \end{cases}$$

Optimal stopping problem:

$$u\left(t, S_t\right) = \sup_{\tau^B \in \mathcal{T}} J(t, S_t, \tau^B).$$

Utility PDE

Proposition (Expected Utility PDE)

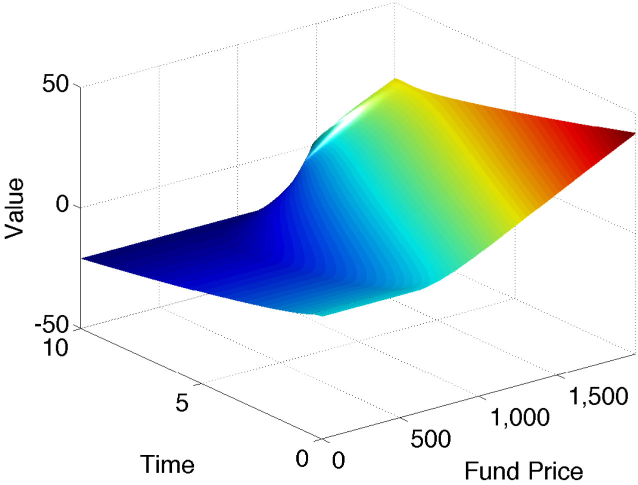
On $\{t < \tau^B \wedge T\}$ the expected utility of the equity-linked life insurance with surrender benefit is $u_t = u(t, S_t)$ where $u : [0, T] \times \mathbb{R}_+ \to \mathbb{R}$ satisfies the obstacle problem

$$\min\left\{-\left(\frac{\partial u}{\partial t}(t,s) + \mu s \cdot \frac{\partial u}{\partial s}(t,s) + \frac{1}{2}\sigma^2 s^2 \frac{\partial^2 u}{\partial s^2}(t,s)\right), u_t - U(e^{r(T-t)}L(t))\right\} = 0$$

with terminal condition $u(T, s) = U(\Phi(s))$, for $s \in \mathbb{R}_+$.

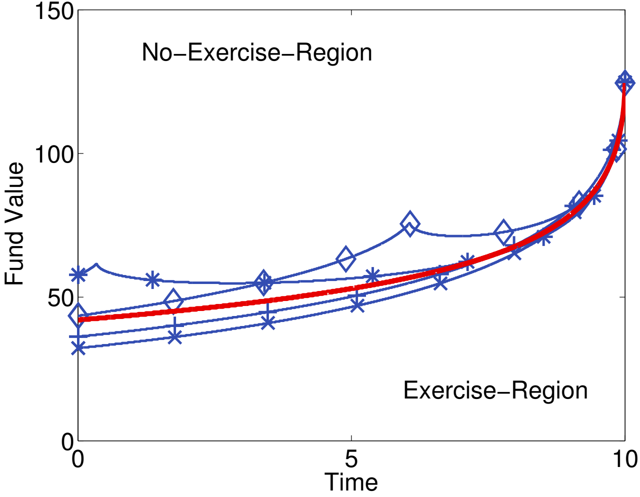
- Viscosity Solution

Expected Utility PDE: loss-averse Preferences

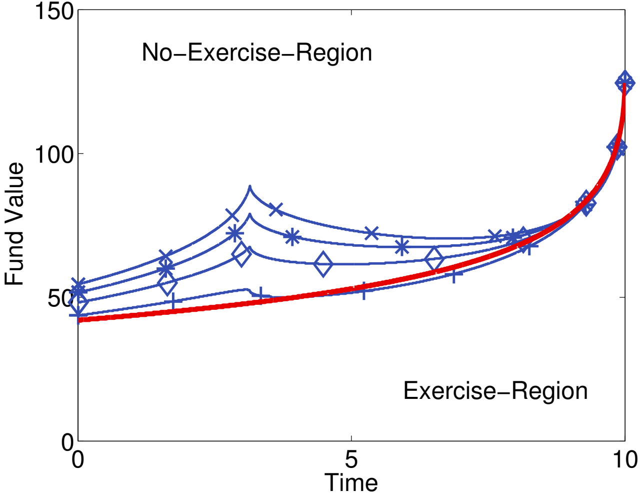


Surrender Decision

Loss-averse Preferences, Different Reference Points



Loss-averse Preferences, Varying Loss Aversion



Conclusion

- Previous insurance literature argues that policyholders either surrender optimally, or treats bounded rationality through exogenous stochastic processes.
- This paper **prices equity-linked life insurance** and **derives the policyholder's surrender behavior from loss-averse preferences**...
- ...and shows that the policyholder's **surrender behavior changes substantially**.