Early Default Risk and Surrender Risk: Impacts on Participating Life Insurance Policies

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Introduction

- Participating life insurance policies provide
  - minimum interest rate guarantee
  - bonus payment (linked to the performance of the issuing company) upon death and/or upon survival
  - very often, also the surrender guarantee, i.e., the payment to the policyholders if they terminate the policies prematurely.

Introduction

- Such policies are defaultable when insurance companies could not pay out the guarantee.
- To protect policyholders from losing too much of their investment, regulators impose early default mechanisms to force the early closure of the companies when necessary.

Introduction

- Both surrender risk and early default risk have impacts on the fair valuation of participating policies.
- Most literature so far has focused on one of the two risk sources but not the interaction between them.
- We take it into account that the early default risk may have influence on surrender risk.

Introduction

Main Contributions:

- Model the influence of early default risk on surrender risk.
- Derive a pricing PDE to characterize the contract value.
- Study the influence of early default risk and surrender risk on contract valuation.
- Study the response of the insurance company to the regulation and to the policyholders' surrender behavior.

Model

- Company Overview: policyholder and equity holder

Assets	Liabilities & Equity
A_0	$L_0 \equiv \alpha A_0$
	$E_0 \equiv (1-\alpha)A_0$

Table 1 : Insurance company's balance sheet at t_0

Model

- Participating Policy
  - survival benefit at maturity date T

$$\Phi(A_T) = L_T^g + \delta\,[\alpha\, A_T - L_T^g]^+ - [L_T^g - A_T]^+, \qquad L_T^g = L_0 e^{g\,T}$$

  - death benefit at death time τ

$$\Psi(\tau, A_\tau) = L_\tau^d + \delta_d\,[\alpha\, A_T - L_T^d]^+ - [L_T^d - A_\tau]^+, \qquad L_\tau^d = L_0 e^{g_d\,T}$$

  - surrender benefit at surrender time λ

$$S(\lambda, A_\tau) = L_\lambda^s - [L_\lambda^s - A_\lambda]^+, \qquad L_\lambda^s = L_0 e^{g_s\,\lambda}$$

Model

- Early Default Mechanism (Grosen and Jørgensen (2002))
  - Default barrier

$$B_t = \theta\, L_0\, e^{gt}, \qquad \theta : \text{default multiplier}$$

  - Early default at time κ

$$\kappa = \inf\{t < T | A_t \leq B_t\}$$

  - Early default benefit at time κ

$$\Upsilon(\kappa, A_\kappa) = \min\{A_\kappa, L_\kappa^g\}.$$

Model

- Stochastic Modeling
  - Company's asset price process A under risk-neutral measure $\mathbb{Q}$

$$\mathrm{d}A_t = r(t)\,A_t\,\mathrm{d}t + \sigma(t, A_t)\,A_t\mathrm{d}W_t^{\mathbb{Q}}, \qquad \forall t \in [0, T]$$

  - Death event
    - Jump process: $H_t = 1_{\{\tau \leq t\}}$
    - Mortality intensity μ: deterministic in time
  - Surrender event
    - Jump process: $J_t = 1_{\{\lambda \leq t\}}$
    - Surrender intensity γ

Model

- Stochastic Modeling: model the surrender intensity γ
  - In most literature, surrender action is considered as an optimal stopping problem.
  - However, contracts are not traded on the market
    - In emergency, policyholders cannot sell the contracts on the market but back to the insurance company.

      $\Rightarrow$low (exogenous) surrender intensity when surrender not optimal
    - Contract values are not observable on the market. Not all policyholders are competent enough to judge the contract values.

      $\Rightarrow$high (endegenous) surrender intensity when surrender optimal

Model

- Stochastic Modeling: model the surrender intensity γ
  - CEIOPS points out that it is necessary to differentiate between different insurance products and different policyholders when modeling the surrender risk.
  - Our approach: similar to Li and Szimayer (QF, 2014) we assume the surrender intensity to be bounded from below by $\underline{\rho} > 0$ and from above by $\bar{\rho} < \infty$.

$$\gamma_t = \begin{cases} \underline{\rho}, & \text{for } S(t,A) < v(t,A) \\ \bar{\rho}, & \text{for } S(t,A) \geq v(t,A) \end{cases}$$

Contract Valuation

Proposition

For $(t, A) \in [0, T) \times \mathbb{R}^+$, the value of the participating policy described above is the solution of the partial differential equation

$$\mathcal{L}v(t,A) + \mu(t)\Psi(t,A) + \gamma_t S(t,A) - (r(t) + \mu(t) + \gamma_t)v(t,A) = 0\,,$$

subject to $v(t, A) = \Upsilon(t, A)$ for $A \leq B_t$, with

$$\gamma_t = \begin{cases} \underline{\rho}\,, & \text{for } S(t,A) < v(t,A)\,, \\ \bar{\rho}\,, & \text{for } S(t,A) \geq v(t,A)\,, \end{cases}$$

and termination condition $v(T, A) = \Phi(A)$, for $A \in \mathbb{R}^+$.

Parameters

Market Parameters		Contract Parameters	
A_0	100	α	85%
r	0.04	T	10
σ	0.2	δ, δ_d	0.9
A^μ	5.0758×10^{-4}	g, d, s	0.02
B	3.9342×10^{-5}		
c	1.1029		
μ	$A^\mu + Bc^{x+t}$		

Table 2 : Parameter specifications

Result 1

Regulation protects limitedly rational but not fully rational policyholders. Overregulation may be bad.

$(\underline{\rho}, \bar{\rho})$	no early default	with early default		
		$\theta = 0.7$	$\theta = 0.9$	$\theta = 1.1$
(0, 0)	85.6141	86.8199	90.4937	89.6619
(0, 0.03)	86.0368	87.0668	90.5088	89.6619
(0, 0.3)	88.1531	88.4680	90.6270	89.6619
$(0, \infty)$	92.0546	92.0548	92.0628	89.6619
(0.03, 0.03)	81.8567	82.9119	86.6744	87.9197
(0.03, 0.3)	84.2656	84.5696	86.8343	87.9197
$(0.03, \infty)$	88.5391	88.5392	88.5436	87.9197
(0.3, 0.3)	75.4561	75.7496	78.0482	83.2947
$(0.3, \infty)$	80.7500	80.7500	80.7500	83.2951

Table 3 : Contract values for different default multipliers θ and different rationality levels represented by $(\underline{\rho}, \bar{\rho})$.

Result 2

Without the early default regulation, the equity holder prefers a less risky investment strategy if the policyholder surrenders optimally; otherwise, a more risky investment strategy may be preferred.

$(\underline{\rho}, \bar{\rho})$	no early default		
	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$
(0, 0)	85.3380	85.6141	84.7199
(0, 0.03)	85.5737	86.0368	85.2578
(0, 0.3)	86.71 56	88.1531	87.9902
$(0, \infty)$	88.3422	92.0546	93.3676
(0.03, 0.03)	82.8209	81.8567	79.7188
(0.03, 0.3)	84.0278	84.2656	83.0419
$(0.03, \infty)$	85.5405	88.5391	89.6150
(0.3, 0.3)	78.2582	75.4561	71.5565
$(0.3, \infty)$	80.7500	80.7500	80.7500

Table 4 : Contract values for different investment strategies represented by σ and different rationality levels represented by $(\underline{\rho}, \bar{\rho})$, $\theta = 0.9$

Result 3

With the early default mechanism, the equity holder has the incentive to reduce the riskiness of the investment, relatively independent of the rationality level of the policyholders.

	with early default		
$(\underline{\rho}, \bar{\rho})$	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$
(0, 0)	86.4204	90.4937	92.4513
(0, 0.03)	86.5152	90.5088	92.4547
(0, 0.3)	87.0566	90.6270	92.4836
$(0, \infty)$	88.3424	92.0628	93.4082
(0.03, 0.03)	83.7463	86.6744	88.1438
(0.03, 0.3)	84.3402	86.8343	88.1934
$(0.03, \infty)$	85.5407	88.5436	89.6439
(0.3, 0.3)	78.4962	78.0482	77.8317
$(0.3, \infty)$	80.7500	80.7500	80.7500

Table 5 : Contract values for different investment strategies represented by σ and different rationality levels represented by $(\underline{\rho}, \bar{\rho})$, $\theta = 0.9$.

Conclusion

- We have modeled the influence of early default regulatory mechanism on surrender risk.
- We have derived a pricing PDE to characterize the contract value and solved it numerically with the finite difference method.
- We have analyzed the influence of early default risk and surrender risk on contract valuation through numerical examples.
- We have analyzed the influence of insurance company's investment strategy on contract valuation and the reaction of the insurance company to the regulation and to the policyholders' rationality level.

The End

Thanks for Your Attention!