Joint Calibration of SPX and VIX option surfaces and applications to pricing and hedging equity and volatility linked hybrid notes

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June 1 2014

Eighth Conference in Actuarial Science and Finance
Samos, Greece

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- This market development opens the door to enabling the hedging of equity volatility hybrid derivatives.
- We develop a model with jumps in the *SPX* price process and joint jumps in the price process and the *VIX*, with negative dependence between the two.
- After a joint calibration a equity-volatility hybrid note is priced and hedged by positions in both options markets and dynamic trading in the equity index and the *VIX* futures contract.

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- The calibration to *VIX* options is conducted by estimating a continuous time finite state Markov chain approximation to the process for the squared *VIX* using the methods of Mijatović and Pistorius (2013).
- The use of a finite state Markov chain for the square of the *VIX* is analytically helpful as one immediately obtains also a finite state continuous time Markov chain for the *VIX* itself by taking the square root of the state vector.

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- The suggested model for the logarithm of the stock is then a stationary process of independent increments scaled by the level of the VIX.
- We adopt such a formulation for the risk neutral process.

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- So if the local volatility of the stock is scaled by the *VIX* the *VIX* of the model will not be the *VIX* of the market.
- We jointly model the *SPX* and the *VIX* risk neutrally as indices and we do this by scaling the local volatility of the *SPX* by the market *VIX* in keeping with time series observations.

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- The model *VIX* is an extraneous computation as no contracts are written on the level of the model *VIX* but only on the market *VIX*.

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- On this path space we consider the pricing and hedging of a locally floored and capped cliquet where the strikes for the floor and cap are adjusted in response to the level of the *VIX*.
- The equity and volatility linked cliquet payout is then hedged with a portfolio of options on both the *VIX* and the *SPX*.

Hedging Exercises

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- The reduction in the ask price made possible by the option hedge is evaluated.
- We further reduce the ask price by introducing dynamic trading in the *SPX* and the *VIX* futures contract.
- The actual dynamic positions are determined on a grid using the *SPSA* (Simultaneous Perturbation and Stochastic Approximation) technique poineered by Spall (1992, 1998).

Squared VIX Dyamics

- Let $v(t)$ be the process for the squared VIX then in each time interval the dynamics has the representation

$$\begin{aligned} dv &= (\alpha_i - \kappa_i v)dt + dM_i(t), \ (t_{i-1} < t \leq t_i), \\ i &= 1, \cdots, n. \end{aligned}$$

where $M_i(t)$ is a positive martingale, α_i, κ_i are the long term mean and mean reversion rates in the i^{th} interval.

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  where $M_i(t)$ is a positive martingale, α_i, κ_i are the long term mean and mean reversion rates in the i^{th} interval.
- Given κ_i equations for the forward squared VIX are solved to determine α_i to match the observed squared forward VIX.

Details on

α_i

- For April 30, 2013 we had 73 options on the *VIX* and 271 options on the *SPX* for all of six maturities of 0.0603, 0.1370, 0.2137, 0.3096, 0.3863, and 0.4630 in years.
- Denote by $\overline{v}(t)$ the expectation of the variance swap at time t. The function $\overline{v}(t)$ solves the the equation

$$d\overline{v} = (\alpha(t) - \kappa\overline{v}(t))\, dt$$

  and for piecewise constant parameters with known values for $\overline{v}(t_i)$, $i = 1, \cdots, n$ we must have

$$\alpha_1 = \kappa_1 \overline{v}(t_1)$$

$$\alpha_{i+1} = \frac{1}{1 - e^{-\kappa_{i+1}(t_{i+1}-t_i)}} \times \left(\kappa_{i+1}\overline{v}(t_{i+1}) - \kappa_{i+1}\overline{v}(t_i)e^{-\kappa_{i+1}(t_{i+1}-t_i)}\right).$$

- These equations define the coefficients α_i given the coefficients κ_i to match the observed initial variance swap curve for the expected

Obtaining the forward squared VIX

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- The expected forward variance swap rates a sample of maturities on April 30 2013 are presented in Table 1.
- The estimated Sato parameters were $\sigma = 0.3818$, $\nu = 0.9314$, $\theta = 0.2659$ and $\gamma = 0.2708$.

Table 1
Forward VIX levels
April 30, 2013

Maturity	Forward Squared VIX
0.0603	0.02213
0.3096	0.03581
0.4630	0.04294

Martingales Driving the VIX

- For each i,let $X_i(t;\sigma_i,\nu_i,\theta_i)$ be the variance gamma process defined by a Brownian motion with drift θ_i and volatility σ_i time changed by a gamma process $G_i(t;\nu_i)$ with unit mean rate and variance rate ν_i.

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- Formally we have for independent Brownian motions $W_i(t)$, also independent of the gamma processes G_i,

$$X_i(t;\sigma_i,\nu_i,\theta_i) = \theta_i G_i(t;\nu_i) + \sigma_i W_i(G_i(t;,\nu_i)).$$

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- Let

$$\omega_i = -\log(E\left[\exp\left(X_i(1)\right)\right]).$$

then

$$M_i(t) = \exp\left(X_i(t;\sigma_i,\nu_i,\theta_i) + \omega_i t\right).$$

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- The parameters of the model for the squared *VIX* are κ_i, σ_i, ν_i and θ_i for $i = 1, \cdots, n$.

VIX Calibration Methodology and Results

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- The calibrated parameter values for a sample of maturities are presented in a Table.

t	κ	σ	ν	θ
.0603	22.6514	2.2158	0.0419	6.7202
.3096	17.2433	3.5393	0.0219	2.4766
.4630	5.6393	1.7057	0.0739	4.1163

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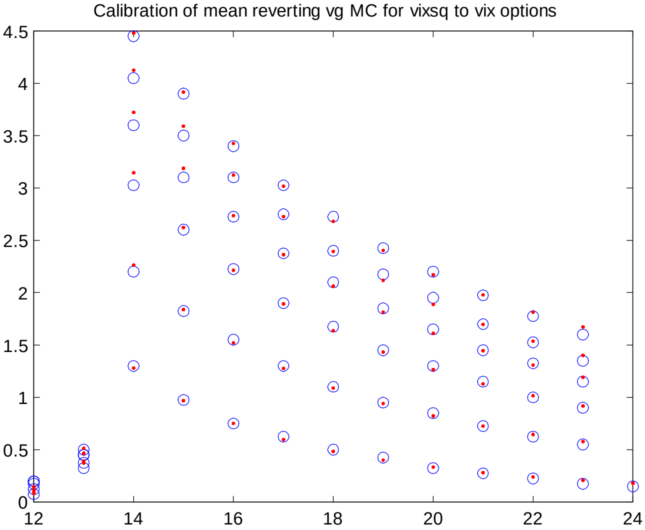
- The fit statistics were

$$\begin{aligned} rmse &= 0.0432 \\ aae &= 0.0326 \\ ape &= 0.0214 \end{aligned}$$


Calibration of mean reverting vg MC for vixsq to vix options
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
12
14
16
18
20
22
24

Implied Long Term VIX Levels

- At these estimated parameter values the solution for coefficients α are presented in Table 3 along with the implied long term *VIX* computed as the square root of α_i/κ_i.

T	.0603	.1370	.2137	.3096	.3863	.4630
α	.5013	.6057	.4791	.6333	.5486	.2800
LTVIX	14.88	16.79	18.57	19.17	20.32	22.28

The SPX as a Variance Gamma Process scaled by the level of the VIX

- Let the level of the VIX at time t be $w(t)$, where

$$w(t) = \sqrt{v(t)}.$$

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- Let the level of the *VIX* at time t be $w(t)$, where

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- We model the risk neutral discounted level of the *SPX* as a martingale driven as a Variance Gamma Lévy process scaled by the *VIX* or by the process $w(t)$.

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- Once again we allow for parameters to be piecewise constant in the intervals $(t_{i-1} < t \leq t_i)$.
- Let $S(t)$ be the stock price at time t with r_i, $\widetilde{q}_i$ the discount rates and dividend yields relevant for the time interval i while $L_i(t)$ is a Variance Gamma Lévy process relevant for the *ith* interval.

The Stock Price Model

- We suppose that

$$\exp\left(-\left(r_i-\widetilde{q}_i\right)\left(t-t_{i-1}\right)\right)A(t_{i-1})S(t)=$$
$$A(t_{i-1})S\left(t_{i-1}\right)\times$$
$$\exp\left(\begin{array}{c}\int_{t_{i-1}}^{t}w(s)dL_i(s)-\\ \int_{t_{i-1}}^{t}\int_{-\infty}^{\infty}\left(e^{w(s)x}-1\right)k_i(x)dxds\end{array}\right)$$

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- The programs for the calibration allow for the parameter shifts.
- On ignoring the parameter shifts we may write for the *SPX* that

$$\exp\left(-(r-q)t\right)S(t) = S(0)\times \exp\left(\begin{array}{c} \int_0^t w(s)dL(s) \\ -\int_0^t\int_{-\infty}^{\infty}\left(e^{w(s)x}-1\right)k\left(x\right)dxds \end{array}\right).$$

Characteristic Function for log Stock

- Let the characteristic function of the process $L(t)$ be $\phi_t(u)$ whereby

$$\begin{aligned} E\left[\exp\left(iuL(t)\right)\right] &= \phi_t(u) \\ &= \exp\left(t\psi(u)\right) \end{aligned}$$

where $\psi(u)$ is the characteristic exponent and for a VG process it is

$$\psi(u) = -\frac{1}{\widetilde{\nu}} \ln\left(1 - iu\widetilde{\theta}\widetilde{\nu} + \frac{\widetilde{\sigma}^2\widetilde{\nu}u^2}{2}\right).$$

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- We may rewrite equation for the stock in terms of this characteristic exponent as

$$
\begin{aligned}
&\exp\left(-(r-q)t\right)S(t) \\
= \;& S(0)\exp\left(\int_0^t w(s)dL(s) - \int_0^t \psi(-iw(s))ds\right).
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- The characteristic function for the logarithm of the dividend adjusted discounted SPX is then the expectation of the exponential of

$$\int_0^t [\psi(uw) - iu\psi(-iw)]'Y(s)ds.$$

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- Let $V_j(t; a) = \Phi(t, e_j; a)$.
- The solution is given by the matrix exponential

$$V(t; a) = \exp\left((diag(ia) + A) (T - t) \right) \mathbf{1}$$

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- Let ρ be a matrix with zero diagonal and let $Y(s) = e_i$ when $w(s) = w_i$ as before to then write

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$$d \ln(S) = \omega dt + w' Y(s) dL - Y(s\_)' \rho Y(s) d\xi_s$$

where ξ_s counts the jumps in the chain.

- We thus allow the jump in $\ln(S)$ to be $-\rho_{ij}$ if VIX jumps from w_i to w_j.
- By taking ρ to be upper triangular we may accomodate no stock response to down jumps in volatility but only a response to upward moves in the VIX.

Characteristic Function for Joint Jumps

- For a fixed argument u the characteristic function $\Phi_{z,t}(u) = (\Phi_{z,i,t}(u)),\ i \in \mathcal{S})$ of $\ln(S(t))$ given that $\ln(S_0) = z$ and $w_0 = w_i$ solves the vector equation

$$\dot{V}_u = \mathcal{L}V$$

subject to

$$V_u(z, 0) = e^{iuz}\mathbf{1}$$

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- We conjecture the solution

$$V(z,t) = e^{iuz}\exp\left(tB(u)\right)\mathbf{1}$$

Characteristic Function for Joint Jumps

- For a fixed argument u the characteristic function $\Phi_{z,t}(u) = (\Phi_{z,i,t}(u)),\ i \in \mathcal{S})$ of $\ln(S(t))$ given that $\ln(S_0) = z$ and $w_0 = w_i$ solves the vector equation

$$\dot{V}_u = \mathcal{L}V$$

subject to

$$V_u(z,0) = e^{iuz}\mathbf{1}$$

- We conjecture the solution

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- and deduce that

$$\begin{aligned} B(u) &= diag(\psi(uw)) + iudiag(\omega) + A \circ R(u) \\ \psi(u) &= -\frac{\sigma^2 u^2}{2} + \int_{-\infty}^{\infty} (\exp(iuy) - 1 - uy)\nu(dy) \\ R(u)_{ij} &= \exp(-iu\rho_{ij}) \end{aligned}$$

Results on SPX and VIX with joint jumps

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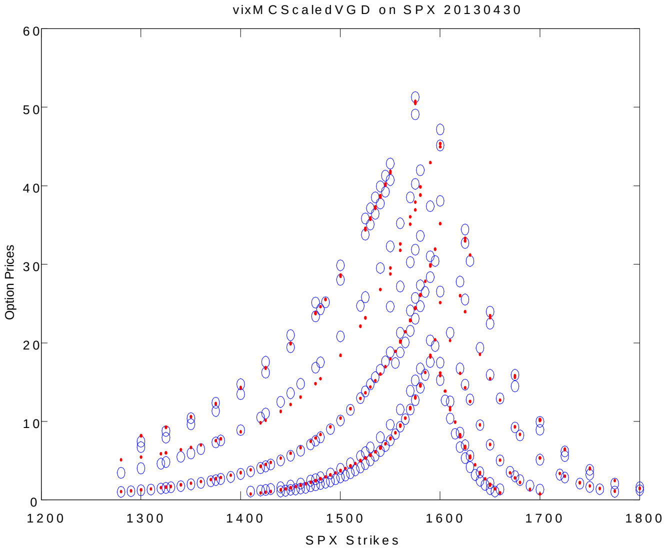
- We report a sample of parameter estimates with η reported in basis points.

Maturity	$\widetilde{\sigma}$	$\widetilde{\nu}$	$\widetilde{\theta}$	η
.0603	.0319	.1479	−1.9289	98
.2137	.6894	87.23	−3.8359	165
.3096	.2477	.2554	−1.7210	8
.4630	.1882	.4309	−0.9774	200


vixMCScaledVGD on SPX 20130430
Option Prices
SPX Strikes
0
10
20
30
40
50
60
1200
1300
1400
1500
1600
1700
1800

Remarks on Simulated Path Spaces

- Given the level of the *VIX* index, v_t at any time t, at one of 200 possible levels in the continuous time Markov chain, one may compute the quadratic variation of the logarithm of the *SPX* for the next 21 days.

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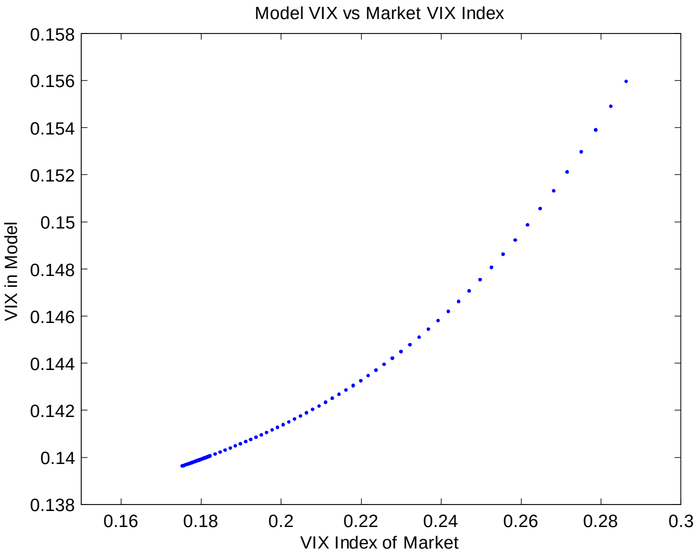
- Here v_i is the level of the *VIX* index in the economy and $\widetilde{v}_i$ is the *VIX* index of the model when the *VIX* index is at level v_i.


Model VIX vs Market VIX Index
0.158
0.156
0.154
0.152
0.15
0.148
0.146
0.144
0.142
0.14
0.138
VIX in Model
0.16
0.18
0.2
0.22
0.24
0.26
0.28
0.3
VIX Index of Market

Gap and Vol of Vol

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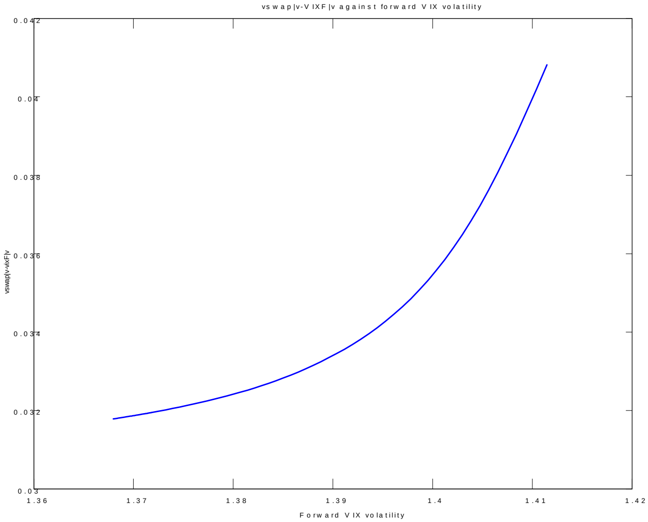
- This gap is expected to be greater, the greater is the volatility of the *VIX* level over the future 21 days.


vswap|v-VIXF|v against forward VIX volatility
0.042
0.04
0.038
0.036
0.034
0.032
0.03
vswap|v-vixF|v
1.36
1.37
1.38
1.39
1.4
1.41
1.42
Forward VIX volatility

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- We consider the perspective of an investor who wishes to exit equity exposures when volatility rises but is willing to have a greater exposure when volatilities are lower.
- These considerations suggest revising the floor and cap in response to the volatility.
- The cash flow to the *SPX* and *VIX* linked cliquet for a notional of N dollars is given by

$$C = N\sum_{t=1}^{12}\left[\begin{array}{c}\left(-\frac{0.5}{\sqrt{50}}\frac{0.2}{v_t}-R_t\right)^{+}+\\ R_t-\left(R_t-\frac{0.5}{\sqrt{50}}\frac{0.2}{v_t}\right)^{+}\end{array}\right].$$

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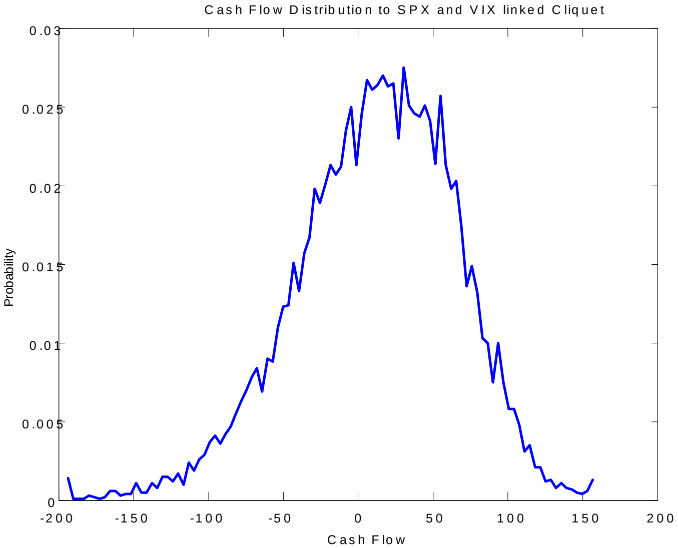
- The ask price for this cash flow at the lower stress levels of 0.25 and 0.5 using the distortion *minmaxvar* are respectively $33,361$ and $51,506$.
- However, it is unreasonable to sell such a security unhedged and expect to be able to successfully charge for it in a competitive market.



Cash Flow Distribution to SPX and VIX linked Cliquet
0.03
0.025
0.02
0.015
0.01
0.005
0
Probability
-200
-150
-100
-50
0
50
100
150
200
Cash Flow

Residual Cash Flows Priced

- The hedge instruments are priced at their risk neutral expectations and the residual cash flow is given by

$$R = \alpha' \left(H - E[H] \right) - C$$

where H is the matrix of cash flows to the 364 hedging assets, $E[H]$ is the cost of accessing these hedge cash flows,α is the position vector in the 364 hedging assets and C is the target cash flow to be paid out.

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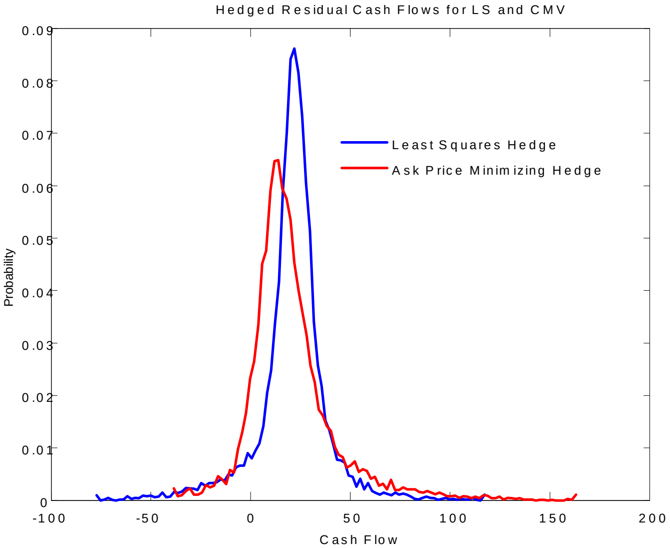
- The ask price for the least squares hedge is $35,616$ while for ask price minimizing hedge the ask price is $32,428$.



Hedged Residual Cash Flows for LS and CMV
Least Squares Hedge
Ask Price Minimizing Hedge
Probability
Cash Flow
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
-100
-50
0
50
100
150
200

Enhanced Delta Hedged Cash Flow

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- We took 25 values for the *SPX* and the *VIX* in this range that we may denote by S_k, V_l for $k.l = 1, \cdots, 25$.

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- We then seek positions a_{kl} and b_{kl} in the *SPX* and the *VIX* furures contract conditional on the *SPX* and the *VIX* being at the levels S_k, V_l.

Delta Hedged Payoff

- The payoff to the delta positioning is

$$\delta_T^i = \sum_{kl} \left[\begin{array}{c} \mathbf{1}_{\left(S_t^i, V_t^i\right) \in B_{kl}} a_{kl} \left(S_T^i - S_t^i\right) + \\ \mathbf{1}_{\left(S_t^i, V_t^i\right) \in B_{kl}} b_{kl} \left(V_T^i - E_t[V_T | V_t^i = V_l]\right) \end{array} \right].$$

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- The level of VIX futures was obtained using a kernel estimator.

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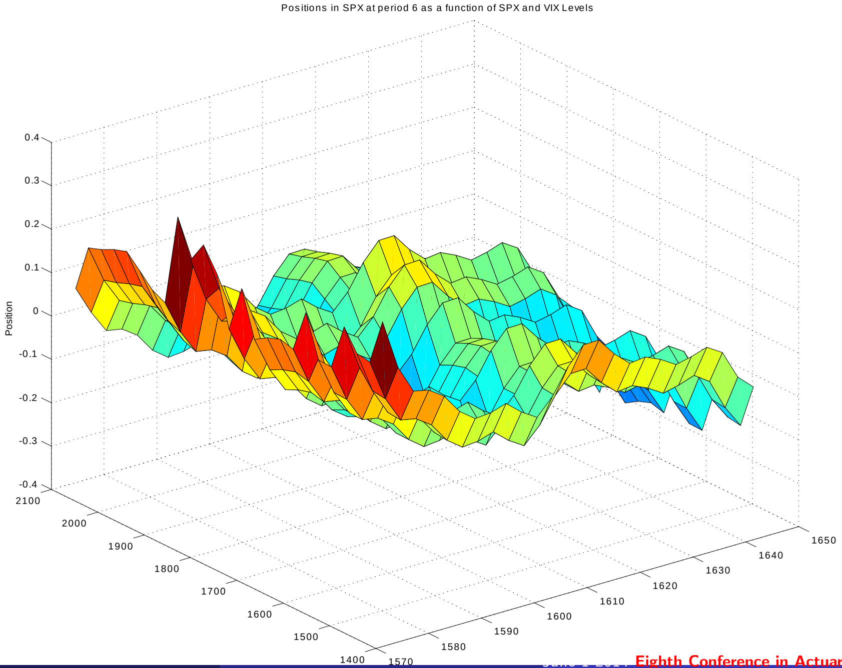
- The *SPSA* optimized ask price was further reduced to $31,849$ at the*minmaxvar* stress level of 0.75.


Positions in SPX at period 6 as a function of SPX and VIX Levels
Position
0.4
0.3
0.2
0.1
0
-0.1
-0.2
-0.3
-0.4
2100
2000
1900
1800
1700
1600
1500
1400
1570
1580
1590
1600
1610
1620
1630
1640
1650

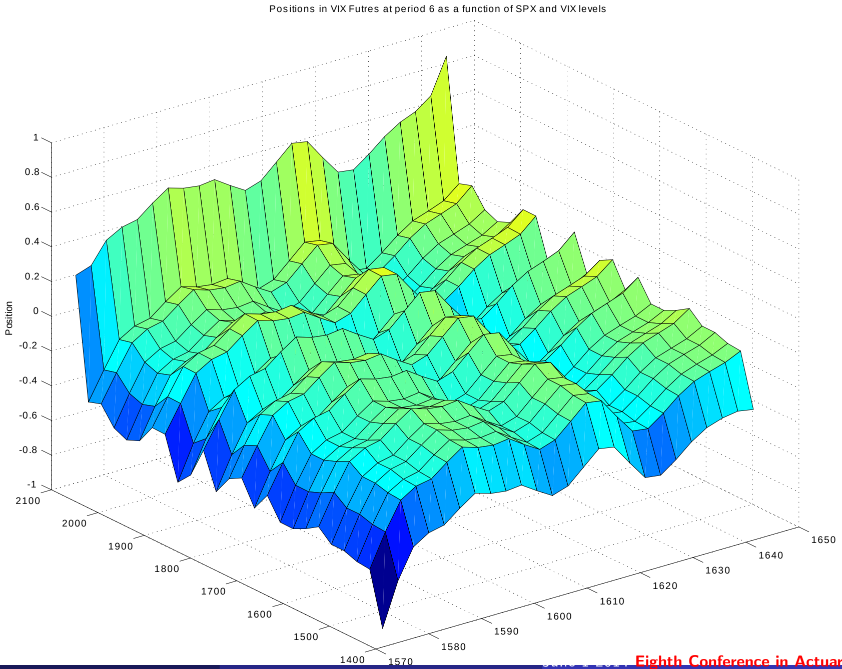

Positions in VIX Futres at period 6 as a function of SPX and VIX levels
Position
1
0.8
0.6
0.4
0.2
0
-0.2
-0.4
-0.6
-0.8
-1
2100
2000
1900
1800
1700
1600
1500
1400
1570
1580
1590
1600
1610
1620
1630
1640
1650

Figure: Position in VIX Futures in 30 days as function of the levels of the SPX and the VIX at this time point.

Conclusion

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