Joint Calibration of SPX and VIX option surfaces and applications to pricing and hedging equity and volatility linked hybrid notes

Dilip B. Madan

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Eighth Conference in Actuarial Science and Finance

Samos, Greece
Introductory Remarks

- An active option market has developed on the CBOE Volatility Index (VIX).

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SPX and VIX hybrids

Eighth Conference in Actuarial
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We develop a model with jumps in the SPX price process and joint jumps in the price process and the VIX, with negative dependence between the two.
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We develop a model with jumps in the SPX price process and joint jumps in the price process and the VIX, with negative dependence between the two.

After a joint calibration a equity-volatility hybrid note is priced and hedged by positions in both options markets and dynamic trading in the equity index and the VIX futures contract.
We model the square of the \( VIX \) that is a variance swap price, as mean reverting and we match the initial variance swap term structure at selected maturities.
Model Structure

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The use of a finite state Markov chain for the square of the VIX is analytically helpful as one immediately obtains also a finite state continuous time Markov chain for the VIX itself by taking the square root of the state vector.
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Motivation for Model Structure

- The modeling for the logarithm of the stock price is inspired by Eberlein, Kallsen and Kristen (2003).
- They observe that among a variety of measures for devolatizing stock returns, the best candidate appears to be the level of the \( VIX \).
- The suggested model for the logarithm of the stock is then a stationary process of independent increments scaled by the level of the \( VIX \).
- We adopt such a formulation for the risk neutral process.
In the adopted formulation, the expected quadratic variation of the logarithm of the stock in the model over a future interval of a month is the expected level of the integral of the future squared VIX for a month.
VIX of the Market and VIX of the Model

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We jointly model the \( SPX \) and the \( VIX \) risk neutrally as indices and we do this by scaling the local volatility of the \( SPX \) by the market \( VIX \) in keeping with time series observations.
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The model VIX is an extraneous computation as no contracts are written on the level of the model VIX but only on the market VIX.
Once the model is jointly calibrated, we simulate joint paths for the \textit{SPX} and \textit{VIX} levels.
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On this path space we consider the pricing and hedging of a locally floored and capped cliquet where the strikes for the floor and cap are adjusted in response to the level of the \textit{VIX}. 

\begin{itemize}
  \item \textit{SPX} and \textit{VIX} hybrids
  \item Dilip B. Madan (Smith School of Business)
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On this path space we consider the pricing and hedging of a locally floored and capped cliquet where the strikes for the floor and cap are adjusted in response to the level of the VIX.

The equity and volatility linked cliquet payout is then hedged with a portfolio of options on both the VIX and the SPX.
The ask price for the cliquet is modeled using distorted expectations as described in Cherny and Madan (2010).
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The reduction in the ask price made possible by the option hedge is evaluated.

We further reduce the ask price by introducing dynamic trading in the SPX and VIX futures contract. The actual dynamic positions are determined on a grid using the SPSA (Simultaneous Perturbation and Stochastic Approximation) technique pioneered by Spall (1992, 1998).
Hedging Exercises

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Let \( v(t) \) be the process for the squared VIX then in each time interval the dynamics has the representation

\[
dv = (\alpha_i - \kappa_i v) dt + dM_i(t), \quad (t_{i-1} < t \leq t_i), \\
i = 1, \ldots, n.
\]

where \( M_i(t) \) is a positive martingale, \( \alpha_i, \kappa_i \) are the long term mean and mean reversion rates in the \( i^{th} \) interval.
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Given \( \kappa_i \) equations for the forward squared VIX are solved to determine \( \alpha_i \) to match the observed squared forward VIX.
Details on $\alpha_i$

- For April 30, 2013 we had 73 options on the VIX and 271 options on the SPX for all of six maturities of 0.0603, 0.1370, 0.2137, 0.3096, 0.3863, and 0.4630 in years.

- Denote by $\overline{V}(t)$ the expectation of the variance swap at time $t$. The function $\overline{V}(t)$ solves the equation

$$d\overline{V} = (\alpha(t) - \kappa \overline{V}(t)) \, dt$$

and for piecewise constant parameters with known values for $\overline{V}(t_i)$, $i = 1, \cdots, n$ we must have

$$\alpha_1 = \kappa_1 \overline{V}(t_1)$$

$$\alpha_{i+1} = \frac{1}{1 - e^{-\kappa_{i+1}(t_{i+1} - t_i)}} \times$$

$$\left( \kappa_{i+1} \overline{V}(t_{i+1}) - \kappa_{i+1} \overline{V}(t_i) e^{-\kappa_{i+1}(t_{i+1} - t_i)} \right).$$

- These equations define the coefficients $\alpha_i$ given the coefficients $\kappa_i$ to match the observed initial variance swap curve for the expected forward variance swap rates.
Obtaining the forward squared VIX

- These values were obtained by calibrating the Sato process based on the variance gamma model as formulated in Carr, Geman, Madan and Yor (2007) to the VIX options as an interpolator.

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<thead>
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<tr>
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<td>0.04294</td>
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Table 1

Forward VIX levels
April 30, 2013

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Obtaining the forward squared VIX

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For each $i$, let $X_i(t; \sigma_i, \nu_i, \theta_i)$ be the variance gamma process defined by a Brownian motion with drift $\theta_i$ and volatility $\sigma_i$ time changed by a gamma process $G_i(t; \nu_i)$ with unit mean rate and variance rate $\nu_i$. Then $M_i(t) = \exp(X_i(t; \sigma_i, \nu_i, \theta_i) + \omega_i t)$. The parameters of the model for the squared VIX are $\kappa_i, \sigma_i, \nu_i$ and $\theta_i$ for $i = 1, 2, \ldots, n$. 
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Formally we have for independent Brownian motions $W_i(t)$, also independent of the gamma processes $G_i$,

$$X_i(t; \sigma_i, \nu_i, \theta_i) = \theta_i G_i(t; \nu_i) + \sigma_i W_i(G_i(t; \nu_i)).$$
Martingales Driving the VIX

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- Let

$$\omega_i = -\log(E[\exp(X_i(1))]).$$

then

$$M_i(t) = \exp(X_i(t; \sigma_i, \nu_i, \theta_i) + \omega_i t).$$
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The parameters of the model for the squared $VIX$ are $\kappa_i$, $\sigma_i$, $\nu_i$ and $\theta_i$ for $i = 1, \ldots, n$. 
The model was calibrated by constructing a 200 point continuous time finite state Markov chain for the squared VIX on a non uniform grid following the methods of Mijatović and Pistorius (2013).
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The calibrated parameter values for a sample of maturities are presented in a Table.

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The fit statistics were

\[
\text{rmse} = 0.0432 \\
\text{aae} = 0.0326 \\
\text{ape} = 0.0214
\]
Calibration of mean reverting vg MC for vixsq to vix options
At these estimated parameter values the solution for coefficients $\alpha$ are presented in Table 3 along with the implied long term $VIX$ computed as the square root of $\alpha_i/\kappa_i$.

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<td>.6057</td>
<td>.4791</td>
<td>.6333</td>
<td>.5486</td>
<td>.2800</td>
</tr>
<tr>
<td>$LTVIX$</td>
<td>14.88</td>
<td>16.79</td>
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The SPX as a Variance Gamma Process scaled by the level of the VIX

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We model the risk neutral discounted level of the SPX as a martingale driven as a Variance Gamma Lévy process scaled by the VIX or by the process $w(t)$. 

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Once again we allow for parameters to be piecewise constant in the intervals $(t_{i-1} < t \leq t_i)$.

Let $S(t)$ be the stock price at time $t$ with $r_i$, $\tilde{q}_i$ the discount rates and dividend yields relevant for the time interval $i$ while $L_i(t)$ is a Variance Gamma Lévy process relevant for the $ith$ interval.
The Stock Price Model

We suppose that

\[
\exp\left(-\left(r_i - \tilde{q}_i\right)(t - t_{i-1})\right) A(t_{i-1}) S(t) = A(t_{i-1}) S(t_{i-1}) \times \\
\exp\left(\int_{t_{i-1}}^{t} w(s) dL_i(s) - \int_{t_{i-1}}^{t} \int_{-\infty}^{\infty} \left(e^{w(s)x} - 1\right) k_i(x) dx ds\right)
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For ease of notation we ignore in this development the parameter shifts occurring at the traded maturities and proceed with the development under constant parameters.
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On ignoring the parameter shifts we may write for the SPX that

$$\exp \left( - (r - q)t \right) S(t) = S(0) \times \exp \left( - \int_0^t \int_{-\infty}^\infty \left( e^{w(s)x} - 1 \right) k(x) \, dx \, ds \right) \left( \int_0^t w(s) \, dL(s) \right).$$
Let the characteristic function of the process $L(t)$ be $\phi_t(u)$ whereby

$$E \left[ \exp \left(iuL(t)\right) \right] = \phi_t(u) = \exp \left(t\psi(u)\right)$$

where $\psi(u)$ is the characteristic exponent and for a VG process it is

$$\psi(u) = -\frac{1}{\tilde{v}} \ln \left(1 - iu\tilde{\theta}\tilde{v} + \frac{\tilde{\sigma}^2 \tilde{v} u^2}{2}\right).$$
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$$\psi(u) = -\frac{1}{\tilde{\nu}} \ln \left( 1 - iu\tilde{\theta}\tilde{\nu} + \frac{\tilde{\sigma}^2\tilde{\nu}u^2}{2} \right).$$

We may rewrite equation for the stock in terms of this characteristic exponent as

$$\exp \left( -(r - q)t \right) S(t) = S(0) \exp \left( \int_0^t w(s)dL(s) - \int_0^t \psi(-iw(s))ds \right).$$
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We may then write

$$w(t) = w' Y(t).$$
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We may then write

$$w(t) = w' Y(t).$$

The characteristic function for the logarithm of the dividend adjusted discounted \textit{SPX} is then the expectation of the exponential of

$$\int_0^t [\psi(uw) - iu\psi(-iw)]' Y(s) ds.$$
Define

\[ \Phi(t, Y(t); a) = E_t \left[ \exp \left( i \int_t^T a' Y(s) ds \right) \right] \]
Matrix Exponentials

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Let \( V_j(t; a) = \Phi(t, e_j; a). \)

The solution is given by the matrix exponential

\[ V(t; a) = \exp \left( (\text{diag}(ia) + A)(T - t) \right) \mathbf{1} \]
Joint jumps may be accommodated by writing

\[ d \ln(S) = \omega dt + wdL - \rho dw \]

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Let \( \rho \) be a matrix with zero diagonal and let \( Y(s) = e_i \) when \( w(s) = w_i \) as before to then write

\[ d \ln(S) = \omega dt + w' Y(s) dL - Y(s_-)' \rho Y(s) d\xi_s \]

where \( \xi_s \) counts the jumps in the chain.
Joint jumps may be accommodated by writing

$$d \ln(S) = \omega dt + wdL - \rho dw$$

where $\omega$ is chosen to get the right exponential drift.

Let $\rho$ be a matrix with zero diagonal and let $Y(s) = e_i$ when $w(s) = w_i$ as before to then write

$$d \ln(S) = \omega dt + w' Y(s) dL - Y(s_-)' \rho Y(s) d\xi_s$$

where $\xi_s$ counts the jumps in the chain.

We thus allow the jump in $\ln(S)$ to be $-\rho_{ij}$ if VIX jumps from $w_i$ to $w_j$. 
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\[ d \ln(S) = \omega dt + w'Y(s)dL - Y(s_-)\rho Y(s)d\zeta_s \]

where \( \zeta_s \) counts the jumps in the chain.

We thus allow the jump in \( \ln(S) \) to be \(-\rho_{ij}\) if VIX jumps from \( w_i \) to \( w_j \).

By taking \( \rho \) to be upper triangular we may accommodate no stock response to down jumps in volatility but only a response to upward moves in the VIX.
For a fixed argument $u$ the characteristic function
\[ \Phi_{z,t}(u) = (\Phi_{z,i,t}(u)), \ i \in S \] of $\ln(S(t))$ given that $\ln(S_0) = z$ and $w_0 = w_i$ solves the vector equation
\[ \dot{V}_u = \mathcal{L} V \]
subject to
\[ V_u(z, 0) = e^{iuz}1 \]
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V_u(z, 0) = e^{iuz} \mathbf{1}
\]
We conjecture the solution
\[
V(z, t) = e^{iuz} \exp(tB(u)) \mathbf{1}
\]
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We conjecture the solution

$$V(z, t) = e^{iuz} \exp(tB(u))1$$

and deduce that

$$B(u) = \text{diag}(\psi(u\lambda)) + iu\text{diag}(\omega) + A \circ R(u)$$

$$\psi(u) = -\frac{\sigma^2u^2}{2} + \int_{-\infty}^{\infty} (\exp(iuy) - 1 - uy)\nu(dy)$$

$$R(u)_{ij} = \exp(-iu\rho_{ij})$$
The specific model with joint jumps that was estimated allowed for

\[ \rho_{ij} = 0, \ j \leq i, \ \text{and} \ \rho_{ij} = \eta \text{ for } j > i \]
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In each of six time intervals there are then four parameters, \( \tilde{\sigma}, \tilde{\nu}, \tilde{\theta} \) and \( \eta \) giving us 24 parameters in all to be estimated by calibration to SPX options.

\begin{tabular}{|c|c|c|c|}
  \hline
  Maturity & \( \tilde{\sigma} \) & \( \tilde{\nu} \) & \( \tilde{\theta} \) & \( \eta \) \\
  \hline
  \hline
  0.603 & 0.0319 & 0.1479 & 1.9289 & 98.2137 \\
  0.6894 & 0.233 & 0.8359 & 165.3096 & 2477.2554 \\
  0.4630 & 0.1882 & 0.4309 & 0.9774 & 200 \\
  \hline
\end{tabular}
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We report a sample of parameter estimates with \( \eta \) reported in basis points.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>( \tilde{\sigma} )</th>
<th>( \tilde{\nu} )</th>
<th>( \tilde{\theta} )</th>
<th>( \eta )</th>
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<tr>
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</tr>
</tbody>
</table>
Remarks on Simulated Path Spaces

Given the level of the \textit{VIX} index, \( v_t \) at any time \( t \), at one of 200 possible levels in the continuous time Markov chain, one may compute the quadratic variation of the logarithm of the \textit{SPX} for the next 21 days.

\[ e_v^i = \frac{\sum_{j=1}^{21} x_t^2 + j v_t = v_i^2}{252} \]

Here \( v_i \) is the level of the \textit{VIX} index in the economy and \( e_v^i \) is the \textit{VIX} index of the model when the \textit{VIX} index is at level \( v_i \).
Remarks on Simulated Path Spaces

- Given the level of the VIX index, $v_t$ at any time $t$, at one of 200 possible levels in the continuous time Markov chain, one may compute the quadratic variation of the logarithm of the SPX for the next 21 days.

- Furthermore one may employ a kernel estimator to compute the conditional expectation of this quadratic variation.
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- We may define $\tilde{v}_i$ as

$$\tilde{v}_i = \sqrt{E_t \left[ \frac{252}{21} \sum_{j=1}^{21} x_{t+j}^2 | v_t = v_i \right]}.$$
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We may also define the expected level of an approximation to the official VIX, in the model as $\overline{v}_i$, where

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By Jensen’s inequality for the square root function we expect a positive gap $g_i$ where

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$$g_i = \tilde{v}_i - \bar{v}_i.$$

This gap is expected to be greater, the greater is the volatility of the VIX level over the future 21 days.
We now present an analysis of a locally floored and capped cliquet that is linked to both the SPX and VIX indices.

For the VIX at 20%, we consider an arithmetic cliquet written on 12 monthly returns capped and floored at 10%. The cap and floor is around twice the monthly volatility of 0.2/\sqrt{12} = 0.0577.

We consider the perspective of an investor who wishes to exit equity exposures when volatility rises but is willing to have a greater exposure when volatilities are lower. These considerations suggest revising the floor and cap in response to the volatility.

The cash flow to the SPX and VIX linked cliquet for a notional of N dollars is given by:

\[ C = N \left( \frac{1}{2} \sum_{t=1}^{12} R_t + \frac{1}{2} \sum_{t=1}^{12} R_t - \frac{3}{5} \right) + \frac{1}{2} \sum_{t=1}^{12} R_t \]
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$$C = N \sum_{t=1}^{12} \left[ \left( -\frac{0.5}{\sqrt{50}} \frac{0.2}{v_t} - R_t \right)^+ + R_t - \left( R_t - \frac{0.5}{\sqrt{50}} \frac{0.2}{v_t} \right)^+ \right].$$
The risk neutral expectation of this cash flow for a million dollar notional on our path space is 11,721 dollars.
Unhedged Ask Price

- The risk neutral expectation of this cash flow for a million dollar notional on our path space is 11,721 dollars.
- We next consider formulating ask prices based on concave distortions as described in Cherny and Madan (2010).
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For a fairly high stress level of 0.75 using the distortion minmaxvar the ask price would be 67,451 and there is only a 9% risk neutral probability of receiving a cash flow greater than this value.
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The ask price for this cash flow at the lower stress levels of 0.25 and 0.5 using the distortion \textit{minmaxvar} are respectively 33,361 and 51,506.
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The ask price for this cash flow at the lower stress levels of 0.25 and 0.5 using the distortion minmaxvar are respectively 33,361 and 51,506.

However, it is unreasonable to sell such a security unhedged and expect to be able to successfully charge for it in a competitive market.
The hedge instruments are priced at their risk neutral expectations and the residual cash flow is given by

\[ R = \alpha' (H - E[H]) - C \]

where \( H \) is the matrix of cash flows to the 364 hedging assets, \( E[H] \) is the cost of accessing these hedge cash flows, \( \alpha \) is the position vector in the 364 hedging assets and \( C \) is the target cash flow to be paid out.
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The ask price for the least squares hedge is 35,616 while for ask price minimizing hedge the ask price is 32,428.
For this purpose we take an intermediate time to be 30 days into the contract or the end of the sixth five day return interval.
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At this time we consider intermediate values for the level of the SPX and the VIX that span the interquartile range of these variates at this time on their simulated marginal distributions.
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At this time we consider intermediate values for the level of the SPX and the VIX that span the interquartile range of these variates at this time on their simulated marginal distributions.

We took 25 values for the SPX and the VIX in this range that we may denote by $S_k, V_l$ for $k,l = 1, \ldots, 25$. 
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We took 25 values for the SPX and the VIX in this range that we may denote by $S_k, V_l$ for $k,l = 1, \ldots, 25$.

We then seek positions $a_{kl}$ and $b_{kl}$ in the SPX and the VIX futures contract conditional on the SPX and the VIX being at the levels $S_k, V_l$. 
Delta Hedged Payoff

- The payoff to the delta positioning is

\[ \delta^i_T = \sum_{kl} \left[ \mathbf{1}(S^i_t, V^i_t) \in B_{kl} a_{kl} (S^i_T - S^i_t) + \mathbf{1}(S^i_t, V^i_t) \in B_{kl} b_{kl} (V^i_T - E_t[V_T | V^i_t = V^i_l]) \right]. \]
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$$\delta^i_T = \sum_{kl} \begin{cases} 1 & (S^i_t, V^i_t) \in B_{kl} \ a_{kl} \ (S^i_T - S^i_t) + \\ 1 & (S^i_t, V^i_t) \in B_{kl} \ b_{kl} \ (V^i_T - E_t[V_T | V^i_t = V^i]) \end{cases}.$$ 

The delta hedged cash flow on path $i$ is

$$\tilde{r}_i = r_i + \delta^i_T.$$
Delta Hedged Payoff

- The payoff to the delta positioning is

\[
\delta^i_T = \sum_{kl} \left[ 1_{(S^i_t, V^i_t) \in B_{kl}} a_{kl} \left( S^i_T - S^i_t \right) + 1_{(S^i_t, V^i_t) \in B_{kl}} b_{kl} \left( V^i_T - E_t[V_T | V^i_t = V_i] \right) \right].
\]

- The delta hedged cash flow on path \( i \) is

\[
\tilde{r}_i = r_i + \delta^i_T.
\]

- The level of VIX futures was obtained using a kernel estimator.
SPSA for Delta Hedge Determination

- We seek position matrices $a_{kl}, b_{kl}$ that minimize the ask price for the residual $\tilde{r}_i$. 

The SPSA optimized ask price was further reduced to 31,849 at the minmaxvar stress level of 0.75.
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This is a problem in dimension $1250 = 2 \times 25 \times 25$. 
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The objective is being accessed on the Monte Carlo path space and it is advisable in such cases to possibly avoid line searches but just to follow steepest descent directions at some adapted multiple.
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This leads us to employ SPSA as described in Spall (1992, 1998) for determining the matrices \( a_{kl}, b_{kl} \).
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Positions in SPX at period 6 as a function of SPX and VIX Levels

SPX

VIX Level

Position

Dilip B. Madan (Smith School of Business)
Figure: Position in VIX Futures in 30 days as function of the levels of the SPX and the VIX at this time point.
Motivated by Eberlein, Kallsen and Kristen (2003) observation that the VIX index is a good way to devolatize the SPX return a risk neutral model is formulated for the logarithm of the SPX as a variance gamma process scaled by the VIX. The square of the VIX is modeled as a finite state continuous time Markov chain and it is calibrated to data on VIX options. Analytical methods are then used to obtain closed forms for the characteristic function of the logarithm of the SPX as a VIX scaled variance gamma process with up jumps in the VIX directly impacting the SPX downwards. The parameters for the risk neutral process for the SPX are then obtained on calibration to SPX options. Sample paths for the calibrated model are simulated and used to comment on the gap between the forward VIX in the model and the expected level of the volatility swap. The simulated path space is also used to price and statically hedge equity and volatility linked notes. The hedge is further enhanced by dynamic trading strategies extracted by an application of SPSA with a view to lowering the ask price based on distorted expectations as described in Cherny and Madan (2010).
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Analytical methods are then used to obtain closed forms for the characteristic function of the logarithm of the \textit{SPX} as a \textit{VIX} scaled variance gamma process with up jumps in the \textit{VIX} directly impacting the \textit{SPX} downwards.

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