Health Insurance and Retirement Incentives

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Introduction

- We study the Optimal retirement problem assuming that
  - the agent chooses the leisure rate during her working period
  - she chooses the (irreversible) retirement date
  - she faces uncertain health shocks
- Before Retirement she has to choose
  - investment strategy
  - consumption rate
  - leisure rate
  - bequest
- After Retirement
  - she fully enjoys leisure
  - she only makes investment-consumption-bequest decisions

Insurances and Retirement Incentives

- Life and Health insurances are available on the market
- We also study the effect of employer provided health insurance (with tied and retiree coverage) on labor and retirement decision
  $\rightarrow$ retirement incentives/disincentives

Literature

Since the seminal contributions of Merton

- Merton (1969) - Lifetime portfolio selection under uncertainty: the continuous time case
- Merton (1971) - Optimum consumption and portfolio rules in a continuous time model

several papers have investigated optimal consumption and portfolio choices in an intertemporal setting.

Literature attempts to adapt the Optimal Strategies problem to real agents'choices studying

1. analytical solutions for Endogenous Retirement problem
2. the calibration of simple models to Real Data: empirical evidence

1- Endogenous Retirement

The literature has addressed the problem assuming a constant wage with a fixed labor rate [1], or an endogenous choice of the labor rate [2,3], or with a stochastic wage but with no flexibility on the labor market [4] or with flexibility (optimal leisure rate) [5].

1. Farhi & Panageas (2007) Saving and Investing for Early Retirement: a Theoretical Analysis
2. Choi & Shim (2006) Disutility, Optimal Retirement, and Portfolio Selection
3. Choi, Shim & Shin (2008) Optimal Portfolio, Consumption-Leisure and Retirement Choice Problem with CES Utility
4. Dybvig & Liu (2005) Lifetime Consumption and Investment: Retirement and Constrained Borrowing
5. Barucci & Marazzina (2012) Optimal investment, stochastic labor income and retirement

2- Empirical evidence

Some contributions

- Life insurance over the life-cyle:
  - Life insurance, Social Security, and household consumption (Hong and Rios-Rull, 2012)
- Labour supply and retirement behavior:
  - Role of medical expenses and insurance (Rust and Phelan, 1997; Blau and Gilleskie, 2006, 2008)
  - Role of health, wealth, and wages (French, 2005)
  - Role of health insurance and Medicare (French and Jones, 2011)
- Why do the elderly save?
  - Role of (uncertainty in) medical expenses, life expectancies (De Nardi et al., 2010).

Common features

1. Health/mortality risk and out-of-pocket medical expenses are important drivers of retirement decisions.
2. No (semi-)explicit solutions: e.g., estimation based on Method of Simulated Moments.
3. Comparative statics difficult: effects of health, wealth, and wages difficult to disentangle.

Our Model

The model aims at clarifying the main trade-offs driving labour supply and retirement decisions under health/mortality risk, while retaining analytical tractability.

- We capture the stylized features of the random environment with a regime-switching framework.
- We derive analytical solutions that can easily be used for comparative statics and policy experiments.
- Casual calibration of the model provides results in line with the empirical evidence on job exit rates and asset allocation in the presence of heterogeneity in life expectancies and medical expenses (e.g., French and Jones, 2011).

The Market

The agent can invest in both the money market and the stock market

For simplicity, we assume that the stock market consists of a single risky asset

Thus we have

- a risk-free instantaneous interest rate r
- a risky asset whose price evolves as

$$dS(t) = bSdt + \sigma SdB(t), \quad S(0) = S_0$$

Consumption, Portfolio & Leisure Processes

We denote by $c(t)$, $\theta_S(t)$ and $l(t)$ the Consumption, Risky Asset Portfolio and Leisure processes, respectively

- $c(t)$ is a nonnegative process
- $l(t) \in [0,1] \cup \{L\}$ is the leisure indicator (rate of leisure at time t)
  1 is the maximum rate of leisure the agent can choose before retirement
- utility function (from consumption and leisure rate)

$$u_{cl}(t,c,l) = \frac{(l(t)^{1-\alpha}\, c^{\alpha}(t))^{1-\gamma}}{\alpha(1-\gamma)}$$

- $\theta_S(t)$ represents the money invested in the risky asset at time t

Let us denote by $\tau_r \geq 0$ the retirement date chosen by the agent; the retirement decision is assumed to be irreversible: during her working life the agent chooses the leisure rate, after retirement the agent fully enjoys leisure time

Therefore

$$0 \leq l(t) \leq 1 \text{ if } 0 \leq t < \tau_r, \quad l(t) = L > 1 \text{ if } t \geq \tau_r$$

Exogenous Health and Aging Shocks

Mortality is described by a Cox process with intensity $\lambda_d(t)$, that is driven by two types of shocks, a health shock and an aging shock

The agent is exposed to an exogenous health shock that occurs at the random time τ_h distributed according to an independent Poisson process of parameter λ_h

The health shock leads to

- Instantaneously medical expenses of size M
- Jump in mortality intensity of size Δ_h
- Wage drop from $\overline{w}$ to $\underline{w}$

$$w(t) := \overline{w} + (\underline{w} - \overline{w})1_{\tau_h \leq t} > 0$$

Before τ_h, the agent's mortality intensity could increase by a jump of size Δ_a at the random time τ_a (aging shock), distributed according to an independent Poisson process of parameter λ_a

Exogenous Health and Aging Shock

- The health deterioration experienced at times τ_a and τ_h is assumed to be irreversible

Denoting by $N_i(t) := 1_{\tau_i \leq t}$ the indicator of the jump at time τ_i, $i \in \{a, h\}$, we can label the resulting four possible health states as

- state "best" (on the event $\{N_a = N_h = 0\}$)
- state "good" (on the event $\{N_a = 1, N_h = 0\}$)
- state "poor" (on the event $\{N_a = 0, N_h = 1\}$)
- state "worst" (on the event $\{N_a = N_h = 1\}$)

The mortality intensity is therefore

$$\lambda_d(t) := \lambda + \Delta_a 1_{\tau_a \leq t} 1_{\tau_a < \tau_h} + \Delta_h 1_{\tau_h \leq t}$$

	On $\{N_h = 0\}$	On $\{N_h = 1\}$
On $\{N_a = 0\}$	$\lambda_d = \lambda$	$\lambda_d = \lambda + \Delta_h$
On $\{N_a = 1\}$	$\lambda_d = \lambda + \Delta_a$	$\lambda_d = \lambda + \Delta_a + \Delta_h$

Insurance

The agent can insure against medical expenses and death risk

- By paying a health insurance premium $\theta_h \lambda_h M$ the agent will receive a benefit of amount $\theta_h M$ in case the health shock (τ_h) occurs

Life Insurance $\rightarrow$ Bequest

- By paying a life insurance premium $\lambda_d(\theta_d - W)$ the agent will receive a benefit $\theta_d - W$ in case death occurs
- When $\theta_d(t) < W(t)$, we interpret the life insurance contract as an annuity
- utility function (from bequest)

$$u_d(t, \theta_d) = \frac{(k_d \theta_d(t))^{\alpha(1-\gamma)}}{\alpha(1-\gamma)}$$

with $k_d > 0$

The Problem

The agent maximizes the expected utility

Given the initial wealth $W(0)$, we look for an admissible strategy that maximizes the objective function

Value Function

$$\mathcal{V}(W(0)) = sup_{\{c,l,\theta_S,\theta_d,\theta_h,\tau_r\}} \; E\left[\int_0^{+\infty} e^{-\int_0^t \beta(s)ds} u(t, c(t), l(t), \theta_d(t))dt\right]$$

- $u(t, c(t), l(t), \theta_d(t)) = u_{cl}(t, c(t), l(t)) + \lambda_d(t) u_d(t, \theta_d(t))$
- $\beta(t) = \delta + \lambda_d(t)$, δ being the subjective discount rate
- subject to the budget constraint

Budget Constraint

The wealth process $W(t)$ (no retirement incentives are considered here) satisfies the dynamic budget constraint

$$
\begin{aligned}
dW(t) &= (1 - N_d(t-))\Big\{ [Y(l(t), w(t)) - c(t)]\, dt \\
&+ \theta_S(t)\,(bdt + \sigma dB(t)) + (W(t) - \theta_S(t))rdt \\
&- (1 - N_h(t-))\lambda_h \theta_h(t) M dt - M(1 - \theta_h(t-))dN_h(t) \\
&- \lambda_d(t)(\theta_d(t) - W(t))dt \Big\} \\
&+ (\theta_d(t-) - W(t-))dN_d(t)
\end{aligned}
$$

Wage process

$$Y(l(t), w(t)) = (1 - l(t)^p)w(t)$$

The Duality Approach

Literature

- He & Pagés (1993) solve via the duality approach the optimal consumption-portfolio problem with a stochastic tradable labor income
- Karatzas & Wang (2000) address a mixed optimal stopping/control problem: the agent chooses the portfolio and the consumption rate and the horizon of the problem deciding the final date as a stopping time
- Choi, Shim & Shin (2008) and Farhi & Panageas (2007) adapt these techniques to address the optimal investment-portfolio problem allowing the agent to choose the retirement date

Post Retirement Problem

The objective function can be written as

$$E\left[\int_0^{\tau_r} e^{-\int_0^t \beta(s)ds} u(t, c(t), I(t), \theta_d(t))dt + e^{-\int_0^{\tau_r} \beta(s)ds} U(\tau_r, W(\tau_r))\right]$$

where $U(\tau_r, W(\tau_r))$ is the optimal expected utility attainable at time τ_r with wealth $W(\tau_r)$

- In τ_r the agent solves the optimal consumption-portfolio-bequest problem with initial wealth $W(\tau_r)$ subject to the budget constraint
- $U(\tau_r, W(\tau_r))$ is the indirect utility function associated with this maximization problem

Dual Utility Functions

- The Convex Conjugate of the utility function $u(t, c, l, \theta_d)$ is

$$\begin{aligned}\widetilde{u}(t,z) &= \max_{c\geq 0\, 0\leq l\leq 1} u_{cl}(t,c,l) - (c + w(t)l^p)z + \lambda_d(t)\max_{\theta_d} u_d(\theta_d) - \theta_d z \\ &:= \widetilde{u}_{cl}(t,z) + \lambda_d(t)\widetilde{u}_d(z)\end{aligned}$$

- The Convex Conjugate of the utility function $U(t, w)$ is

$$\widetilde{U}(\tau, z) = \sup_{w\geq 0} U(\tau, w) - wz$$

Both $\widetilde{u}(t, z)$, and $\widetilde{U}(\tau, z)$ can be computed analytically.

The Dual Problem

Dual objective function

$$E\Big[\int_0^{\tau_r} e^{-\int_0^t \beta(s)ds} \Big\{ \widetilde{u}(t, z(t)) + z(t) \Big(w(t) - \lambda_h e^{-\lambda_h t} M \Big) \Big\} dt + e^{-\int_0^{\tau_r} \beta(s)ds} \widetilde{U}(\tau_r, z(\tau_r)) \Big]$$

where

- $z(t) = \lambda e^{\int_0^t \beta(s)ds} H(t), \quad z(0) = \lambda$
- λ being a Lagrange Multiplier
- $H(t) := e^{-\left(r+\frac{1}{2}\Theta^2\right)t - \Theta Z(t)} e^{-\int_0^t \lambda_d(s)ds}$ is the state-price-density process
- $\Theta := \frac{b-r}{\sigma}$ is the market price of financial risk

The dual problem is solved analytically

Casual Calibration

French E., Jones J.B., *The effects of health insurance and self-insurance on retirement behaviour*, Econometrica, 79-3, 2011

The authors estimate a life-cycle model for 60-year old agents with different risk preferences, wealth/wages, and health characteristics.

- We present numerical results based on casual calibration of the model to Health and Retirement Study (HRS) data and preference estimates provided in French&Jones (2011)
- The HRS is a sample of individuals aged 51-61 in 1992, together with their spouses, who have been surveyed every two years since then
- We focus on male households and look at the optimal decisions made by a 60 years old from today onwards
- we use preference parameters based on the Preference Index developed by French&Jones (2011).

Numerical Results

- French&Jones (2011) use three questions from the HRS to estimate a person's "willingness" to work. The index is discretized into three values: *high*, *low*, and *out*.

In this talk we provide results for:

Agent A (*high*)

- $\delta = 0.056$; $\alpha = 0.412$; $\gamma = 16.752$
- $\overline{w} = 2.6$; $\underline{w} = 1.8$; $k_d = 2.5$
- $\lambda = 0.033$; $\Delta_a = 0.03$; $\Delta_h = 0.05$
- $\lambda_a = 0.2$; $\lambda_h = 0.125$
- $M = 7.57$; $W_0 = 15$

Average Agent

- $\delta = 0.030$; $\alpha = 0.550$; $\gamma = 9.691$
- $\overline{w} = 3$; $\underline{w} = 2$; $k_d = 2.5$
- $\lambda = 0.033$; $\Delta_a = 0.03$; $\Delta_h = 0.05$
- $\lambda_a = 0.2$; $\lambda_h = 0.125$
- $M = 7.57$; $W_0 = 27.9$

W_0 and $\overline{w}$, $\underline{w}$ are given in 10000 USD, and are affected by a taxation of wealth and wage of 33% and 40%, respectively

Moreover, we set

- market parameters: $r = 0.03$; $b = 0.06$; $\sigma = 0.2$

and

- $L = 1.2$

leisure increases by 20% at retirement

- $p = 2$

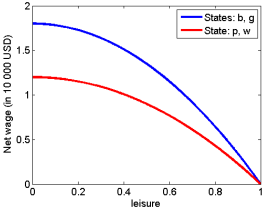


Figure: Net wage as a function of leisure for the 'average' agent

switching from full time to half time (i.e., increasing leisure from zero to 0.5) would generate a 25% drop in gross income

in line with Aaronson & French (2004)

Retirement Decision

The agent's optimal retirement τ_r^* coincides with the first time $t \geq 0$ such that the optimal wealth exceeds the health state dependent threshold.
The health shock determines a switch to a lower threshold target.

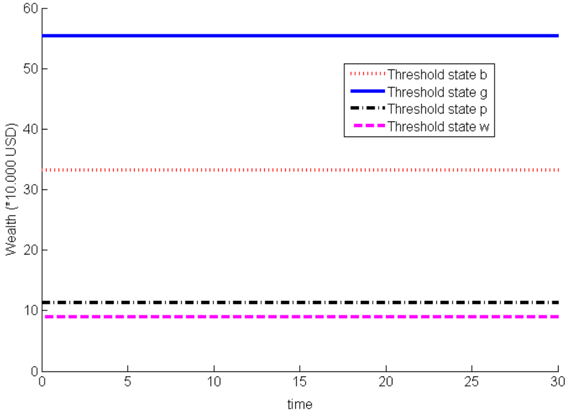


Figure: Retirement thresholds for the 'average' agent (net wealth)

Optimal Strategies

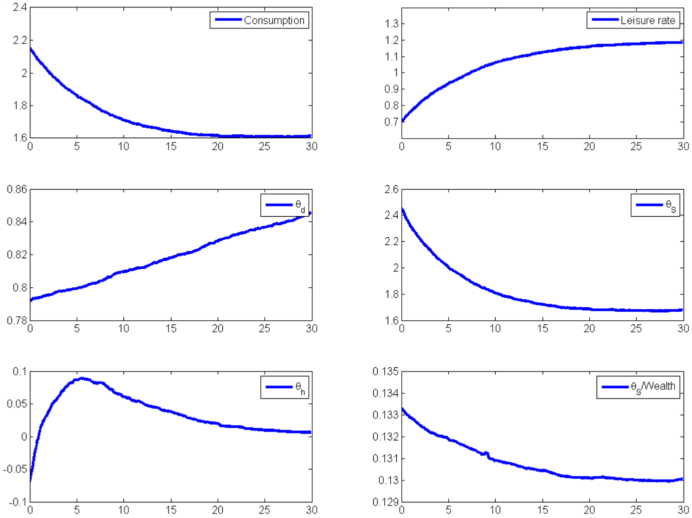


Figure: Optimal strategies for the 'average' agent. Average values computed on the basis of 1000 simulations for the state variables and a time horizon of 30 years. Expected retirement date: $\tau_r^* = 8.02$.

Pre-Retirement Optimal Strategies

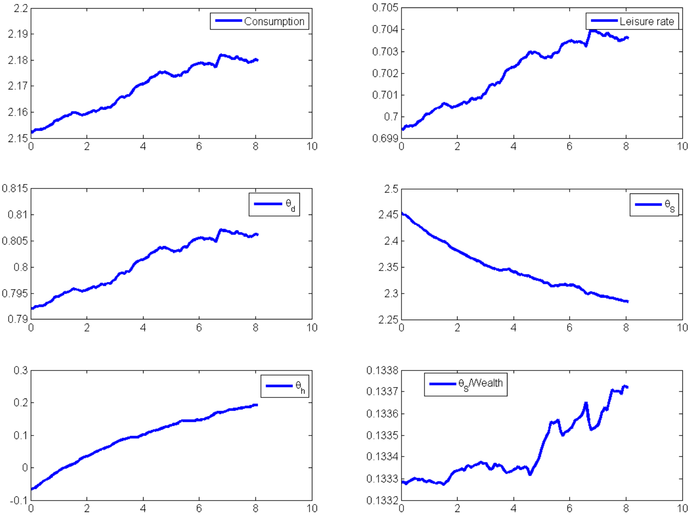


Figure: Optimal strategies (before retirement) for the 'average' agent. Average values computed on the basis of 1000 simulations for the state variables, considering each simulation until retirement occurs. Expected retirement date: $\tau_r^* = 8.02$.

Pre-Retirement Optimal Strategies

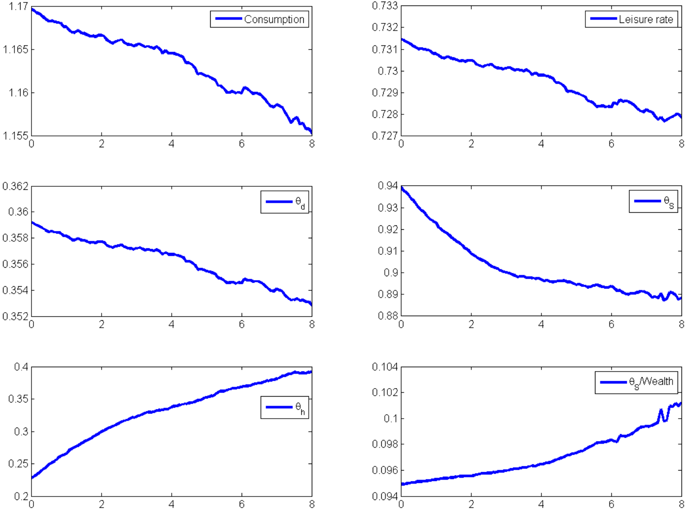


Figure: Optimal strategies (before retirement) for agent A. Average values computed on the basis of 1000 simulations for the state variables, considering each simulation until retirement occurs. Expected retirement date: $\tau_r^* = 7.94$.

Agent A

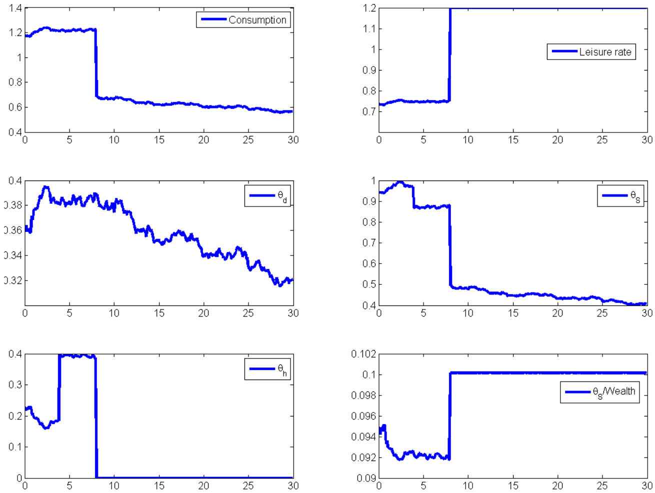


Figure: One simulation: aging shock ($b \to g$) occurs after 3.8748 years and retirement occurs after 7.9775 years, due to the health shock ($g \to w$).

Agent A

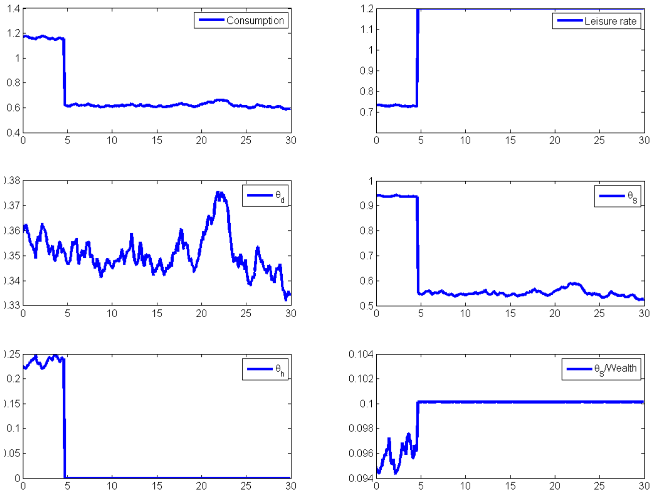


Figure: One simulation: no aging shock occurs, retirement occurs after 4.6667 years, due to the health shock ($b \to p$).

Retirement Incentives

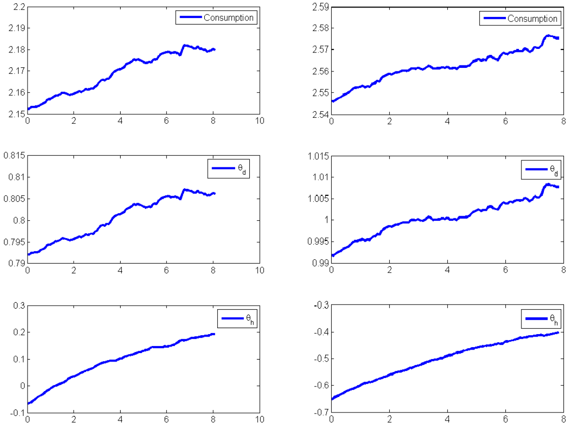
In addition to the baseline model, where only private health insurance is available and is decided endogenously, we consider three forms of health insurance:

- Health coverage is provided by the employer while the agent is actively working (*tied* coverage).
- Health coverage is provided by the employer also while the agent is retired (*tied-retiree* coverage).
- Health coverage is provided by the employer when the agent is retired (*retiree* coverage).

Health coverage pays the medical expenses M when the health shock occurs.

Optimal Strategies (before retirement)

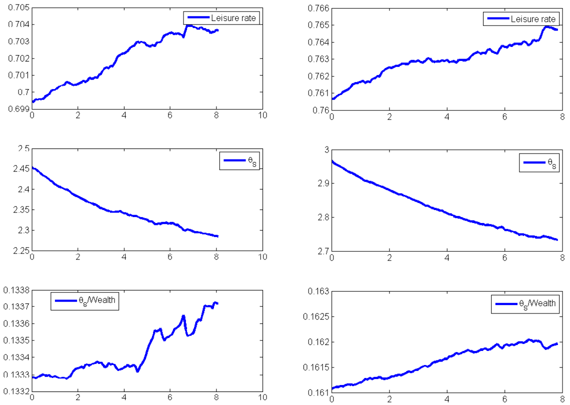
'Average' agent: without (left) / with (right) tied coverage



- The availability of tied coverage increases consumption.
- The negative optimal allocation to health insurance in the case of tied coverage suggests that the agent is overinsured and is trying to undo the employer provided coverage.

Optimal Strategies (before retirement)

'Average' agent: without (left) / with (right) tied coverage



- The availability of tied coverage reduces working hours before retirement.
- Employer provided health insurance leads to greater stock market participation.

Retirement Decision in presence of Health Coverage

Health insurance	b	g	p	w
No insurance	33.2541	55.3052	11.2605	8.9887
tied	94.5995	176.3530	11.2605	8.9887
retiree	7.3863	6.8324	11.2605	8.9887
tied-retiree	18.7913	20.89558	11.2605	8.9887

Table: Retirement thresholds for the 'average' agent

- The availability of retiree coverage induces an immediate retire
- As expected, the retirement thresholds for the after shock states (p and w) do not change
- The presence of *tied* coverage induce the agent to postpone her retirement decision, while the presence of a *retiree* coverage, even if coupled with a *tied* one, accelerates retirement with respect to the baseline model without insurance

Conclusions

- We obtain distributions of job exit times that are in line with the empirical evidence (and expected retirement between ages 67-68, depending on individual characteristics and insurance availability).
- If we consider optimal strategies beyond retirement, the model yields a jump in consumption at the retirement date, whose size is consistent with the empirical evidence.
- The patterns and levels of the optimal investment/insurance/consumption strategies are overall in line with the extant empirical literature (e.g., French, 2005).
- Work in progress includes extended numerical analysis (comparative statics), estimation of the model on PSID data, and the design of relevant policy experiments (e.g., age-dependent health insurance constraints/subsidies, like Medicare).