The Impact of Stochastic Volatility and Policyholder Behaviour on Guaranteed Lifetime Withdrawal Benefits

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- Client pays premium P into account which is invested in and performing with some reference asset(s) and can withdraw each year a portion without charge until death
- "GLWB" rider guarantees "withdrawals" even if account is depleted
- Policyholder can surrender the contract paying surrender fees on the amount exceeding the penalty-free level

- Several versions of enhancements: roll-up / various ratchet riders
- Fees: guarantee fee (periodic), management fee (periodic), upfront fee
- More flexible setup: policyholder is entitled to withdraw any amount from the account, subject to penalty charges (partial surrender)

- Policyholder behaviour influences substantially value of the contract

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- Common model is some form of suboptimal surrender - varying widely
- Resale on secondary markets common in some contries (US, UK), less in others (Germany, Austria)
- Discussion about introduction of obligatory notice in Germany (akin to UK)

Behavioral Models

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Assume client withdraws either the penalty-free amount or everything - in-between amounts found to be suboptimal

Contract enhancement riders

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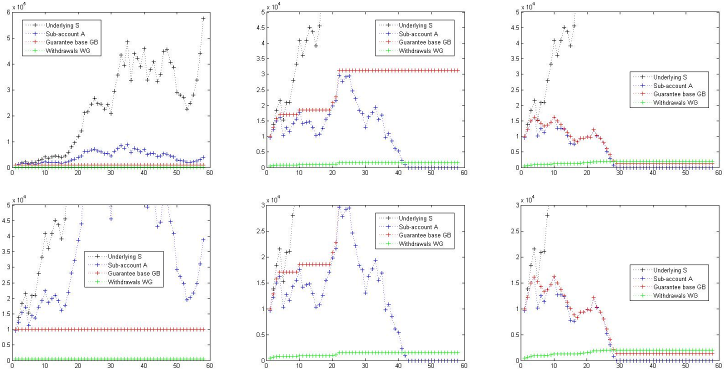
  - No ratchet - $WG_t = WG_0$ (NOR)

Ratchet Mechanisms in PLIs



a) NOR mode **b) NDB mode** **c) RB mode**

Evolution of accounts in a bullish scenario for all ratchet modes; top: larger scales; bottom: zoomed in

Asset dynamics

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2. **Stochastic volatility model, Heston (1993):**

$$\begin{aligned} dS(t) &= rS(t) + \sqrt{V(t)} S(t) dW_S^Q(t) \\ dV(t) &= \kappa(\theta - V(t))dt + \xi \sqrt{V(t)} dW_V^Q(t), V(0) \geq 0, \end{aligned}$$

κ - weight on mean-reversion; θ - long-term volatility average; ξ - "vol of vol"; W_S^Q, W_V^Q independent Q-Brownian Motions correlated by ρ; market price $\gamma(t)$ of volatility risk restricted to be linear in volatility, $\gamma(t) = \sqrt{V(t)}$

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- backwards in time from a maximum possible age, approximating the continuation value by regressing realized continuation values from payments in $\{t+1, \dots\}$ on state variables at each withdrawal date t and surrendering if it is higher than the current surrender value
- extensible to arbitrary withdrawal levels allowed (RIP)

Numerical Results

Guarantee trigger time $\tau = min(t : A_t < WG_t)$:
First time the subaccount A_t falls under the penalty-free limit (absorbing state)

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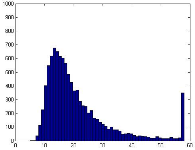
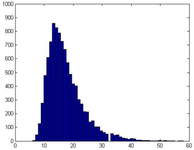
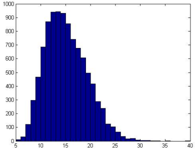


No ratchet



NDB ratchet



RB ratchet

Distribution of guarantee ITM start times

For more enhanced riders, distribution is shifted to shorter trigger times

Determination of fair parameters

Literature approaches for the penalty-free withdrawal rate:

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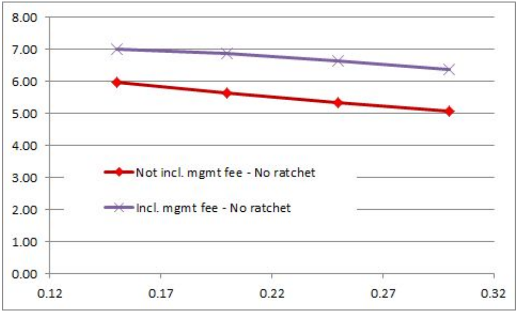
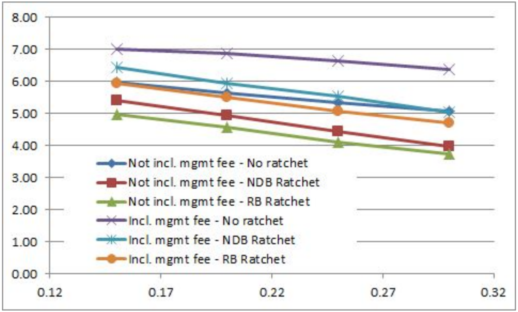


volatility

Fair initial penalty-free withdrawal rate in percent by volatility with alternatives managed and unmanaged fund (No Ratchets, No Surrender)



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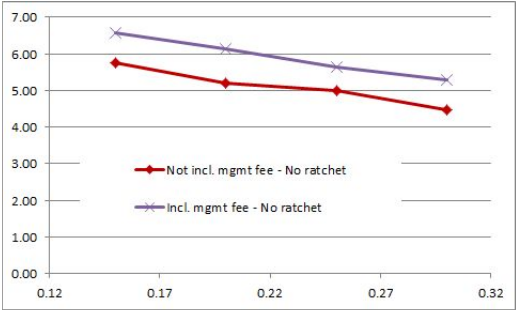
Fair initial penalty-free withdrawal rate in percent with alternatives managed and unmanaged fund

No, NDB and RB Ratchets, No Surrender



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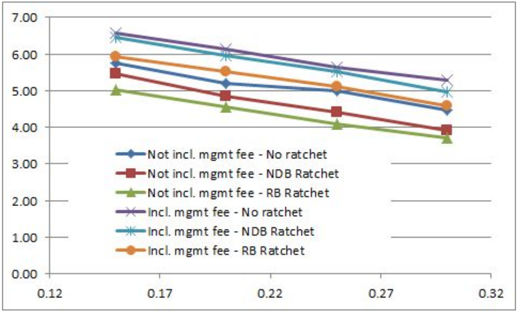

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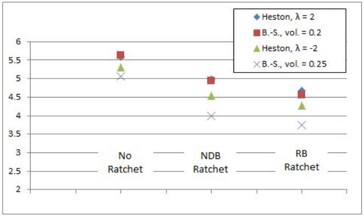
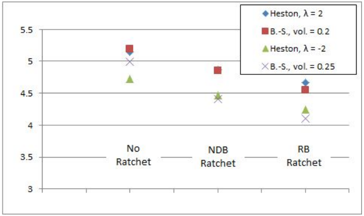
Comparing GBM and Stochastic Volatility under the Heston model



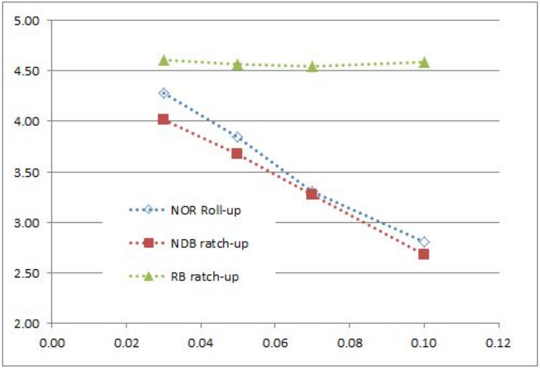
No surrender



Optimal Surrender

Impact of stochastic volatility under the Heston model on the fair initial penalty-free withdrawal rate is rather modest

Impact of roll-up of the guaranteed withdrawal rate on the fair rate

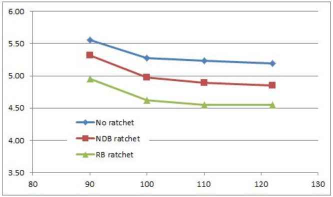


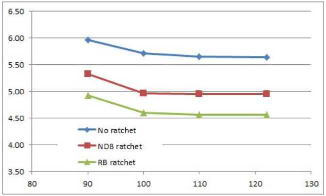
Fair withdrawal rate under Optimal Surrender and different ratchet modes

Impact of roll-up on the fair withdrawal rate is substantial for No ratchet and NDB ratchet

Effect of ratchet more important than roll-up in the RB mode

Sensitivity of fair initial penalty-free withdrawal rate on limiting age





Fair withdrawal rate in % under Optimal Surrender by limiting age

Impact of the limiting age on the fair withdrawal rate is small for values above 100

Suboptimal Behaviour: Based on Moneyness of the Guarantee

The client surrenders at time t with probability

$p_m(M_t) = max(a(M_t), 0)$, where

Moneyness M_t defined as $M_t = \frac{f(Y_t)}{I(t,Y_t)}$, with

$f(Y_t)$: surrender value
$I(t, Y_t) = \sum_{i=t+1}^{\omega} e^{-r(i-t)}\ WG_i\ \tau_i^D$: present value of an annuity paying the current penalty-free amount of WG_t

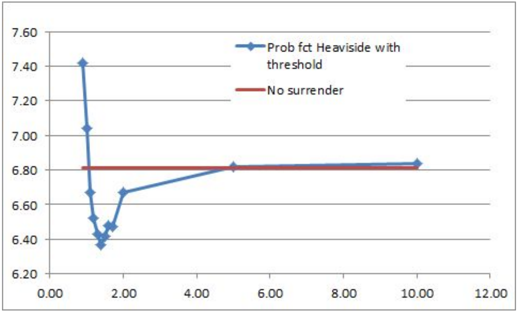
ω: maximum possible age

τ_i^D: Probability of death in $(i, i+1]$

a: function allowing imperfectness in this decision

Smoothing model for $a(x)$: eg.

- $a_1(x) = \mathbb{H}(x - b)$: all remaining clients surrender when the moneyness reaches threshold b



Fair initial penalty-free withdrawal rate under surrender based on moneyness with $a = a_1(x)$, by threshold b

GBM, No Ratchets, $\sigma = 0.2$

- $a_2(x) = 1 - e^{-(x-b)}$: smooth transition from continuing to surrender

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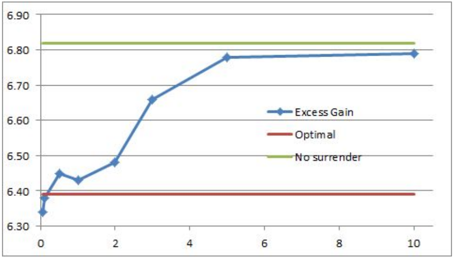
- $a_3(x) = p_t^D \eta(x)$, p_t^D deterministic, with

$$\eta(x) = \begin{cases} 1/3 & \text{if } x < 0.95, \\ 1 & \text{if } 0.95 \leq x < 1.05, \\ 3 & \text{if } 1.05 \leq x < 1.15, \\ x & \text{if } x > 1.15. \end{cases}$$

Year	Surrender rate
1	6%
2	5%
3	4%
4	3%
5	2%
6	1%

Suboptimal Behaviour: Based on Excess Gain from Optimal Behaviour

The client withdraws at the penalty-free level unless the additional benefit from fully surrendering exceeds some factor C of WG_t.
Special cases: $C = \infty$: corresponds to No surrender; $C = 0$ corresponds to Optimal Surrender.



Fair withdrawal rate in % for suboptimal behaviour measuring excess gain from behaving optimally

Suboptimal Behaviour: Based on Secondary Market or Moneyness

The client is informed with probability $p_2(t)$ about existence of a secondary market and an offer of a resale value $L(Y_t) = f(Y_t) + \kappa(V_t^{opt} - f(Y_t)$, where

$V_t^{opt} = max(f(Y_t), \mathbf{E}[V_t^C | Y_t])$: value under optimal surrender,
$\kappa \in [0, 1)$: fraction of the premium of the resale price over the surrender value (Hilpert, Li & Szimayer (2012)).

- If she is not informed, she behaves as in model 3) and decides by moneyness
- If she is informed, she sells the policy to the secondary market with probability $p_m(\frac{L(Y_t)}{I(t, Y_t)})$, with p_m using a smoothing function as in the moneyness model

Summary and Outlook

- Described Lifetime withdrawal guarantees in PLIs
- Analyzed various models for policyholder behaviour
- Fair values highly sensitive to other parameters and contract details
- LSMC to be extended for arbitrary withdrawal levels

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Thank you!