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Faculty of Science

Mortgages *-from a household perspective*

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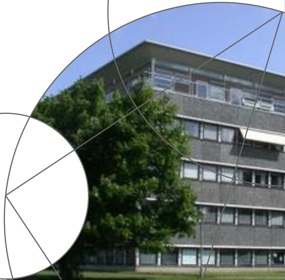

May 31, 2014

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Agenda

- Short introduction to the Danish mortgage market
- Two Danish mortgage products described in a simple setting
- Optimal investment/consumption viewed in relation to mortgages
- Concluding remarks and future research

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The Danish mortgage market is unique in size and structure and is undergoing change

- **Size:** In 2010 the Danish bond market was the second largest in Europe and Danish households had a gross debt 3 times larger than disposable income (primarily in mortgages)
- **Structure:** Danish mortgage system is based on a pass-through principle that match funding and lending to the households 1:1
- **Products:** The market has in less than a decade changed from primarily fixed rate mortgages (FRM) to adjustable rate mortgages (ARM)

www.nykredit.com/investorcom/ressourcer/dokumenter/pdf/Danish_covered_bond_web.pdf
www.nationalbanken.dk/C1256BE2005737D3/side/BBF9D80727ACD8ADC1257966004DE179/file/husholdningernes_balancer_del1.pdf

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Two Danish mortgage products

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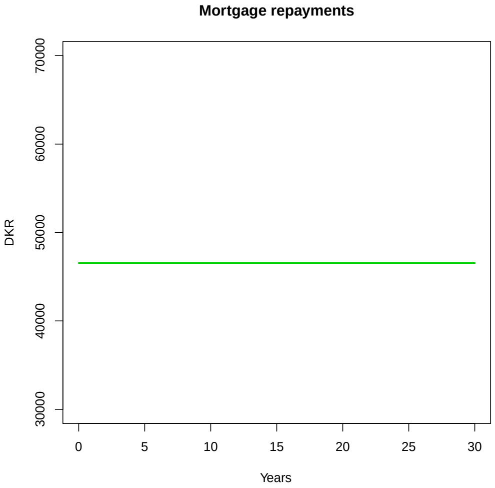
The most simple mortgage payment, is achieved when the mortgage is constructed as an annuity



In a complete bond market, the fixed rate on the annuity is det. by the price of an annuity bond

With the price of an annuity bond $AN^T(0)$ given by the market, the constant interest rate $\bar{r}$ is uniquely determined so that the initial principal of the mortgage $\text{Princ}_0^{\text{FRM}}$, satisfy:

$$\text{Princ}_0^{\text{FRM}} = \bar{b} \cdot AN^T(0)$$

with

$$\bar{b} = \frac{\text{Princ}_0^{\text{FRM}}}{\frac{1}{\bar{r}}\left(1 - e^{-\bar{r}\cdot T}\right)}.$$

In DK the most common ARM is created by determining a fixed rate for a shorter period n

At every refinancing node $t_i = i \cdot n < T$, the constant interest until next node is determined through an annuity bond with termination at the next node, $AN^{t_{i+1}}(t_i)$, and the price of a ZCB, $P^{t_{i+1}}(t_i)$, so that:

$$\text{Princ}_{t_i} = \hat{b}_i \cdot AN^{t_{i+1}}(t_i) + \text{Princ}_{t_{i+1}} \cdot P^{t_{i+1}}(t_i),$$

where $\text{Princ}_{t_{i+1}} = \hat{b}_i \frac{1}{\hat{r}_i}\left(1 - e^{-\hat{r}_i(T-t_{i+1})}\right)$ denotes the outstanding principal at time t_{i+1} of the annuity amortization of interest $\hat{r}_i$ and with

$$\hat{b}_i = \frac{\text{Princ}_{t_i}}{\frac{1}{\hat{r}_i}\left(1 - e^{-\hat{r}_i(T-t_i)}\right)}.$$

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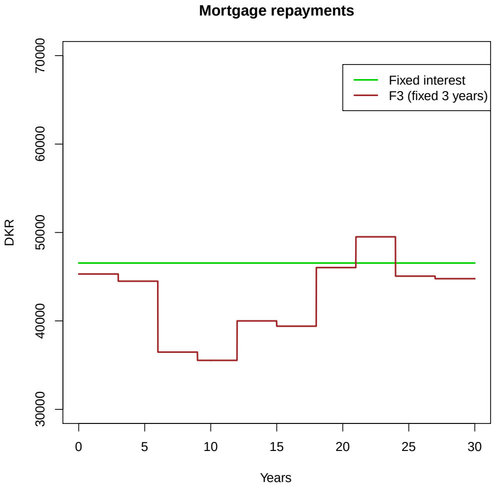
The payments of the F*n*-loan are easily simulated under assumption of Vasicek interest



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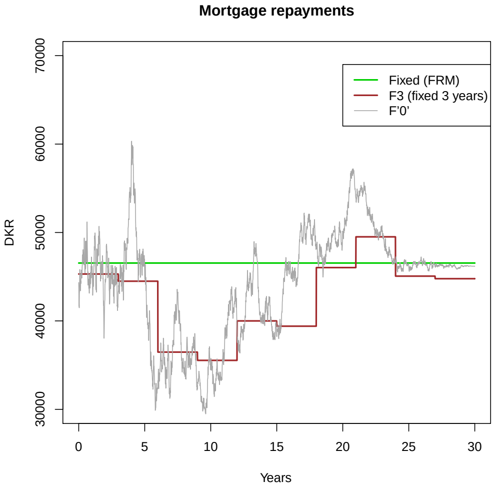
In a continuous-time setting, it is convenient to consider the ARM with infinite refinancing



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The limit ARM is eqv. to 100% in the bank & payments cont. det. by short-rate annuity-factor

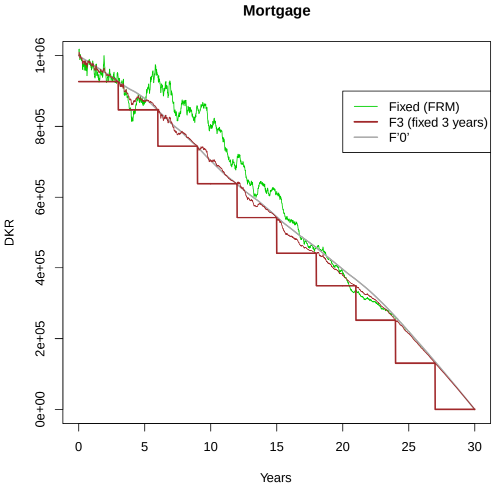
Under the Vasicek assumption, the market-value of the loan in the limit follows the dynamics:

$$dMV^{\text{ARM}_\infty}(t) \quad = \quad r(t)MV^{\text{ARM}_\infty}(t)dt - \hat{b}_\infty(t)dt,$$

with payments given by:

$$\hat{b}_\infty(t) \quad = \quad \frac{MV^{\text{ARM}_\infty}(t)}{\frac{1}{r(t)}\left(1 - e^{-r(t)(T-t)}\right)}.$$

It is noted that the market-values of the products in a sense behaves oppositely to the payments



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Optimal investment/consumption

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Introduction of labour income allows for optimal investment/consumption w/ negative initial wealth

Assume income perfectly correlated with the bond, through:

$$dY(t) = [\bar{\mu} + br(t)]Y(t)dt + \bar{\sigma}_Y Y(t)dW^P(t)$$

It is straight forward to value the payment stream of labour in the Vasicek market, thereby defining the Human Capital

$$\text{Human Capital} = E_t^Q\left[\int_t^T e^{-\int_t^s r du} Y(s)ds\right] = Y(t)F(t, r(t))$$

(Human Capital originally introduced by Merton et al 1992, and with the above labour income a simple version of Munk/Kraft 2011)

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Optimal investment/consumption has an explicit solution in the simple setting

Under the assumption of Vasicek interest, it is possible to find the solution to:

$$\sup_{\pi,c} E_t\left[\int_t^T e^{-\delta(u-t)}\frac{1}{1-\gamma}c_u^{1-\gamma}du\right]$$

through application of HJB. The optimal investment/consumption is derived to be on the form:

$$\begin{aligned}
c^*(t) &= \frac{X(t)+Y(t)F(t,r)}{g(t,r)} \\
\pi^*(t) &= \frac{X(t)+Y(t)F(t,r)}{X(t)}\left(\frac{1}{\gamma}\frac{\lambda_B}{\sigma_B(t)}+\frac{\sigma_r}{\sigma_B(t)}\frac{Y(t)F_r(t,r)}{X(t)+Y(t)F(t,r)}\right. \\
&\quad \left.-\frac{\sigma_r}{\sigma_B(t)}\frac{g_r(t,r)}{g(t,r)}-\frac{\sigma_Y}{\sigma_B(t)}\frac{Y(t)F(t,r)}{X(t)+Y(t)F(t,r)}\right)
\end{aligned}$$

for some function g and with $Y(t)F(t,r)$ denoting human capital.

(A simple version of the solution in Munk/Kraft 2011)

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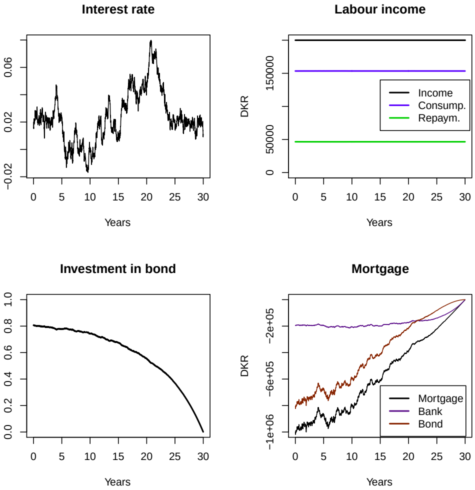
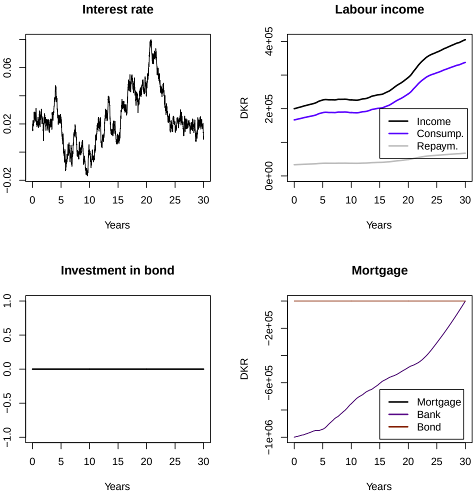
The infinitely risk-averse investor with constant income, prefers the FRM presented earlier



Risk-seeking investor w/ income increasing by interest prefers ARM-like if mkt.price on risk is 0



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Defining mortgage payments as income less consumption, yields a GBM in very limited cases

Defining the optimal mortgage payments as:

$$P^*(t) = Y(t) - c^*(t)$$

gives the payment dynamics:

$$\begin{aligned} dP^*(t) &= \left([\bar{\mu} + br(t)]\, Y(t) - \frac{1}{\gamma}\left[r(t) + \frac{1}{2}(\frac{1}{\gamma}+1)\lambda_B^2 - \delta \right] c^*(t) \right) dt \\ &\quad + \left[\bar{\sigma} Y(t) - \frac{1}{\gamma}\lambda_B c^*(t) \right] dW^P(t) \end{aligned}$$

which is only a generalized geometric Brownian motion in the cases where the coefficients equal.

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In conclusion, it is not straight-forward to link an investor to the mortgages offered in the market

- The two most popular products on the Danish mortgage market are both quite extreme
- Assumptions on labour income heavily influence the optimal mortgage
- In a very simple setting, it is hence difficult to match consumer preferences to one particular mortgage product

Future research

- How does the general power investor optimally combine the existing products?
- Given a particular mortgage product, what is then the optimal investment/consumption?
- What are the preferences of an investor who might want/need to exit the market at a random point in time?

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Primary References

- H. Kraft, C. Munk. Optimal Housing, Consumption, and Investment Decisions over the Life Cycle. Management Science, 2011.
- Z. Bodie, R. C. Merton, W. F. Samuelson. Labour supply flexibility and portfolio choice in a life cycle model. Journal of Economic Dynamics and Control, 1992.
- C. Munk, C. Sorensen. Optimal Consumption and Investment Strategies with Stochastic Interest Rates. Journal of Banking and Finance, 2004.
- R. Korn, H. Kraft. A stochastic control approach to portfolio problems with stochastic interest rates. Journal on Control and Optimization, 2002.