Pricing and Hedging Options in Energy Markets by Black-76

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based on a joint work with F.E. Benth

Energy market

- We consider
  - electricity spot $S(t)$
  - forward and futures contracts on power

$$f(t, T) = \mathbb{E}_Q[S(T) \,|\, \mathcal{F}_t]$$

  - plain vanilla call and put options on the forward and futures

$$C(t, \tau, T) = \mathbb{E}_Q\left[\max(f(\tau, T) - K, 0) \,|\, \mathcal{F}_t\right] .$$

- Fix a pricing measure Q, state dynamics under this pricing measure
- Set $r = 0$

- Consider options on forwards in energy markets
- Two models for the Spot (underlying of the forward)
  1. Black 76 framework

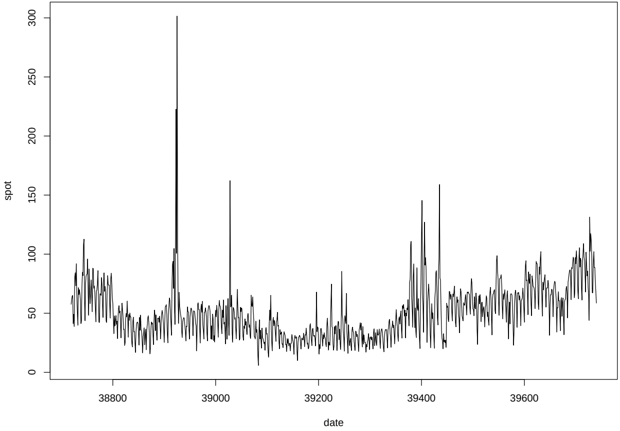

Figure: Phelix base load spot prices from 02.01.2006 until 19.10.2008

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  1. Black 76 framework
  2. Two factor model

- Consider options on forwards in energy markets
- Two models for the Spot (underlying of the forward)
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  2. Two factor model
- Quantify the difference in the corresponding option prices
- Show that big, fast mean reverting spikes do not influence the option price significantly
- Argumentation for Black 76 formula

Intuition: Delivery Period

- Energy markets: futures contracts delivering the underlying energy over a specified period
- short-term shocks in the spot may vanish in the futures dynamics due to smoothing by the delivery period.
- short-term factor of the forward price evolution inherited from the spot might be insignificant in the option price

Outline

- Two factor model for the spot and implied forward prices
- Option price formula and quantification of the pricing error
- Conclusion

Two Factor Spot Price Model

exponential 2-factor model after Gibson and Schwartz [4]

$$S(t) = \Lambda(t) \exp(X(t) + Y(t)) \,.$$

- deterministic seasonality function $\Lambda(t) : \mathbb{R}_+ \to \mathbb{R}_+$
- non-stationary long-term factor X is a drifted Brownian motion

  $$dX(t) = \mu \, dt + \sigma \, dB(t) \,,$$

  with B being a Brownian motion and $\mu, \sigma > 0$ constants.
- stationary short-term factor Y is given by the Ornstein-Uhlenbeck dynamics

  $$dY(t) = -\beta Y(t) \, dt + dL(t) \,,$$

  where L is a pure jump Lévy process and $\beta > 0$ a constant.

Notiation and Assumption on L

Notation

- Lévy measure ℓ
- logmoment generating function

$$\phi(\theta) = \ln \mathbb{E}[\exp(\theta L(1))]\,,$$

Assumption

- assume that the Lévy process has finite exponential moments up to order 3

Two Factor Model for the Spot

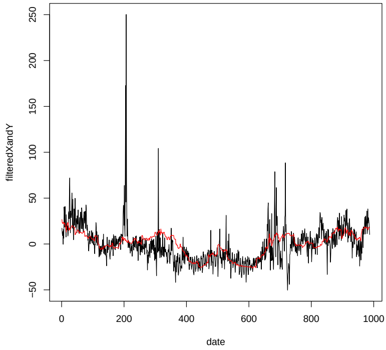

Figure: Deseasonalised EEX electricity spot prices from 02.01.2006 - 19.10.2008

Forward Price dynamics

Proposition 1

The dynamics of the process $t \mapsto f(t,T)$ *for* $t \leq T$ *is*

$$\frac{df(t,T)}{f(t-,T)} = \sigma\, dB(t) + \int_{\mathbb{R}} \left\{ \exp\left(z \mathrm{e}^{-\beta(T-t)} \right) - 1 \right\} \widetilde{N}(dz, dt)\,,$$

where $f(t-,T)$ *denotes the left-limit of* $f(t,T)$*.*

Pricing Call Options on Forwards

The no-arbitrage price of a call option at time $t \leq \tau$ written on a forward contract with price dynamics given as in Prop. 1, is defined by

$$C(t, \tau, T, x) = \mathbb{E}_Q \left[\max(f(\tau, T) - K, 0) \,|\, f(t, T) = x \right] .$$

$\tau \leq T$ exercise time of the call option, with a strike price $K > 0$.

- analyse this price in relation to the Black-76 formula!

Black 76 Formula

Proposition 2

Suppose the forward price dynamics is a geometric Brownian motion

$$\frac{df(t,T)}{f(t,T)} = \sigma\, dB(t)\,.$$

Then the price at time t of a call option with strike K and exercise time $t \le \tau \le T$, *is given by* $C_{B76}(t, f(t,T))$ *with*

$$C_{B76}(t,\tau,T,x) = x\Phi(d_1(x)) - K\Phi(d_2(x))$$

for Φ *being the cumulative standard normal distribution function, and*

$$d_1(x) = d_2 + \sigma\sqrt{\tau - t}$$
$$d_2(x) = \frac{\ln\left(\frac{x}{K}\right) - \frac{1}{2}\sigma^2(\tau - t)}{\sigma\sqrt{\tau - t}}\,.$$

Option Price with Underlying Two Factor Model

Proposition 3

The price of a call option on the forward given in Prop. 1 is

$$
\begin{aligned}
&C(t,\tau,T,x) \\
&\quad = x\mathbb{E}\left[\exp\left(\int_t^\tau \mathrm{e}^{-\beta(T-s)}\,dL(s) - \int_t^\tau \phi(\mathrm{e}^{-\beta(T-s)})\,ds\right)\Phi\left(d_1\left(x,\int_t^\tau \mathrm{e}^{-\beta(T-s)}\,dL(s)\right)\right)\right] \\
&\qquad - K\mathbb{E}\left[\Phi\left(d_2\left(x,\int_t^\tau \mathrm{e}^{-\beta(T-s)}\,dL(s)\right)\right)\right]
\end{aligned}
$$

where $\phi(x)$ is the logarithmic moment generating function of $L(1)$ and

$$
\begin{aligned}
d_1(x,v) &= d_2(x,v) + \sigma\sqrt{\tau - t} \\
d_2(x,v) &= \frac{\ln\left(\frac{x}{K}\right) + v - \int_t^\tau \phi(\mathrm{e}^{-\beta(T-s)})\,ds - \frac{1}{2}\sigma^2(\tau - t)}{\sigma\sqrt{\tau - t}}.
\end{aligned}
$$

Pricing Call Options on Forwards

Theorem 4

Suppose that $\tau \leq T$. Then it holds that

$$\sup_{x\geq 0} |C(t,\tau,T,x) - C_{B76}(t,\tau,T,x)| \leq ((c_{3.6}+c_{3.7})x + (c_{3.4}+c_{3.5})K)\,\mathrm{e}^{-\beta(T-\tau)},$$

for constants

$$c_{3.4} = \frac{1}{\sqrt{2\pi}\sigma}\left(\phi''(0) + \frac{1}{\beta}(\phi'(0))^2\right),$$

$$c_{3.5} = \frac{1}{\sqrt{2\pi\beta}\sigma}\left(\int_{|z|<1} z^2\,\ell(dz) + \int_{|z|\geq 1} \mathrm{e}^{2|z|}\,\ell(dz)\right),$$

$$c_{3.6} = \frac{1}{\sqrt{2\pi}\sigma}\left(\frac{1}{2}\sqrt{\beta} + \frac{1}{\sqrt{\beta}}\left(\mathrm{e}^{1}\int_{|z|<1} z^2\,\ell(dz) + \int_{|z|\geq 1}\mathrm{e}^{3|z|}\,\ell(dz)\right)\right),$$

$$c_{3.7} = c_{3.5}.$$

Forward - Delivery Period

- Typically: forwards delivering the underlying energy over a delivery period $[T_1, T_2]$
- forward price defined as expected average spot over the delivery period: no analytical closed form solution when the spot model is exponential
- choose as delivery time some point in the delivery period, e.g. mid-point $T = (T_1 + T_2)/2$

Pricing Call Options on Forwards

- electricity markets: many options have exercise time equal to the beginning of delivery T_1 of the underlying forward, e.g. $\tau = T_1$.
- relatively long delivery period: τ is relatively far from T^*.
- for a reasonably strong mean reversion β of the spikes, options on forwards in electricity markets can be priced with a high degree of accuracy by the Black-76 formula.

Numerical example - Logarithmic pricing error

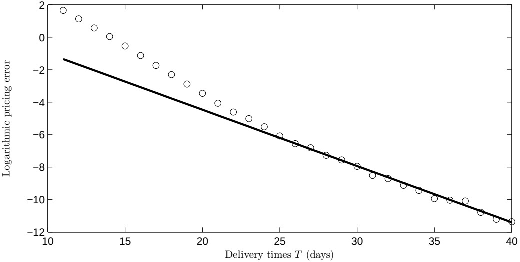

Figure: Difference of the option price to the Black-76 on log-scale. The solid line is a fitted line with slope $-\beta$.

Sensitivity with respect to β

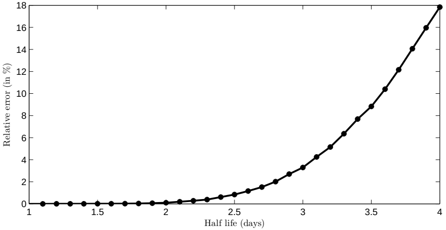

Figure: Relative pricing error in % as a function of the half life of the spike component with 5 spikes on average during a month. $T = 25$

Quadratic Hedging of Call Options on Forwards

- determine the quadratic hedging strategy for our market model
- quadratic hedge converges uniformly to the simple delta-hedging strategy
- rate of convergence same as for the price

Conclusion

- Based on a two-factor spot price model we show that
  - option prices converge exponentially to the Black-76 price in terms of
    - the **speed of mean-reversion** of the stationary factor in the spot price
    - the **time left to maturity** of the forward from the exercise time of the call.
- Combining
  - Approximation the delivery period by its midpoint
  - typically high speed of mean-reversion

  call options on electricity and gas forwards may be priced reasonably accurately by the Black-76 formula in many cases.
- Analogous results for quadratic hedging

How many mean-reverting components?

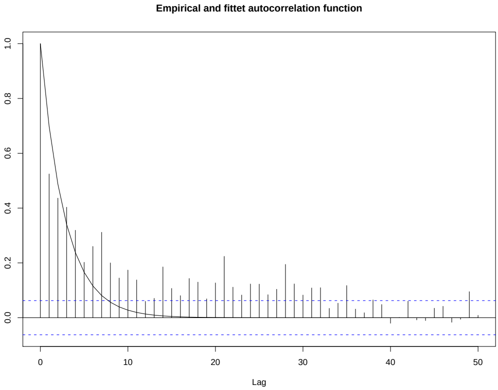

Figure: Empirical and fitted autocorrelation function of a filtered Y in an arithmetic model. $\beta = 0.359$

Empirical Analysis - Illiquidity Issue

Figure: Phelix Option, EEX

Thank you for your attention!

Literature

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