Pricing and Hedging Options in Energy Markets by Black-76

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based on a joint work with F.E. Benth
Energy market

- We consider
  - electricity spot $S(t)$
  - forward and futures contracts on power

$$f(t, T) = \mathbb{E}_Q [S(T) \mid \mathcal{F}_t]$$

- plain vanilla call and put options on the forward and futures

$$C(t, \tau, T) = \mathbb{E}_Q \left[ \max(f(\tau, T) - K, 0) \mid \mathcal{F}_t \right].$$

- Fix a pricing measure $Q$, state dynamics under this pricing measure
- Set $r = 0$
Consider options on forwards in energy markets

Two models for the Spot (underlying of the forward)

1. Black 76 framework
Figure: Phelix base load spot prices from 02.01.2006 until 19.10.2008
Consider options on forwards in energy markets

Two models for the Spot (underlying of the forward)

1. Black 76 framework
2. Two factor model
Consider options on forwards in energy markets

Two models for the Spot (underlying of the forward)

1. Black 76 framework
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Quantify the difference in the corresponding option prices

Show that big, fast mean reverting spikes do not influence the option price significantly

Argumentation for Black 76 formula
Intuition: Delivery Period

- Energy markets: futures contracts delivering the underlying energy over a specified period
- Short-term shocks in the spot may vanish in the futures dynamics due to smoothing by the delivery period.
- Short-term factor of the forward price evolution inherited from the spot might be insignificant in the option price.
Outline

- Two factor model for the spot and implied forward prices
- Option price formula and quantification of the pricing error
- Conclusion
Two Factor Spot Price Model

exponential 2-factor model after Gibson and Schwartz [4]

\[ S(t) = \Lambda(t) \exp(X(t) + Y(t)). \]

- deterministic seasonality function \( \Lambda(t) : \mathbb{R}_+ \rightarrow \mathbb{R}_+ \)
- non-stationary long-term factor \( X \) is a drifted Brownian motion
  \[ dX(t) = \mu \, dt + \sigma \, dB(t), \]
  with \( B \) being a Brownian motion and \( \mu, \sigma > 0 \) constants.
- stationary short-term factor \( Y \) is given by the Ornstein-Uhlenbeck dynamics
  \[ dY(t) = -\beta Y(t) \, dt + dL(t), \]
  where \( L \) is a pure jump Lévy process and \( \beta > 0 \) a constant.
Notation and Assumption on L

Notation

▶ Lévy measure $\ell$
▶ logmoment generating function

$$\phi(\theta) = \ln \mathbb{E}[\exp(\theta L(1))],$$

Assumption

▶ assume that the Lévy process has finite exponential moments up to order 3
Two Factor Model for the Spot

Figure: Deseasonalised EEX electricity spot prices from 02.01.2006 - 19.10.2008
Forward Price dynamics

**Proposition 1**

The dynamics of the process \( t \mapsto f(t, T) \) for \( t \leq T \) is

\[
\frac{df(t, T)}{f(t-, T)} = \sigma \, dB(t) + \int_{\mathbb{R}} \left\{ \exp \left( ze^{-\beta(T-t)} \right) - 1 \right\} \tilde{N}(dz, dt),
\]

where \( f(t-, T) \) denotes the left-limit of \( f(t, T) \).
Pricing Call Options on Forwards

The no-arbitrage price of a call option at time $t \leq \tau$ written on a forward contract with price dynamics given as in Prop. 1, is defined by

$$C(t, \tau, T, x) = \mathbb{E}_Q \left[ \max(f(\tau, T) - K, 0) \mid f(t, T) = x \right].$$

$\tau \leq T$ exercise time of the call option, with a strike price $K > 0$.

- analyse this price in relation to the Black-76 formula!
Black 76 Formula

Proposition 2

Suppose the forward price dynamics is a geometric Brownian motion

\[
\frac{df(t, T)}{f(t, T)} = \sigma dB(t).
\]

Then the price at time \( t \) of a call option with strike \( K \) and exercise time \( t \leq \tau \leq T \), is given by \( C_{B76}(t, f(t, T)) \) with

\[
C_{B76}(t, \tau, T, x) = x\Phi(d_1(x)) - K\Phi(d_2(x))
\]

for \( \Phi \) being the cumulative standard normal distribution function, and

\[
d_1(x) = d_2 + \sigma \sqrt{\tau - t}
\]

\[
d_2(x) = \frac{\ln \left( \frac{x}{K} \right) - \frac{1}{2} \sigma^2 (\tau - t)}{\sigma \sqrt{\tau - t}}.
\]
Option Price with Underlying Two Factor Model

**Proposition 3**

*The price of a call option on the forward given in Prop. 1 is*

\[
C(t, \tau, T, x) = x \mathbb{E} \left[ \exp \left( \int_t^\tau e^{-\beta(T-s)} \, dL(s) - \int_t^\tau \phi(e^{-\beta(T-s)}) \, ds \right) \Phi \left( d_1 \left( x, \int_t^\tau e^{-\beta(T-s)} \, dL(s) \right) \right) \right] - K \mathbb{E} \left[ \Phi \left( d_2 \left( x, \int_t^\tau e^{-\beta(T-s)} \, dL(s) \right) \right) \right]
\]

*where \( \phi(x) \) is the logarithmic moment generating function of \( L(1) \) and*

\[
d_1(x, \nu) = d_2(x, \nu) + \sigma \sqrt{\tau - t}
\]

\[
d_2(x, \nu) = \ln \left( \frac{x}{K} \right) + \nu - \int_t^\tau \phi(e^{-\beta(T-s)}) \, ds - \frac{1}{2} \sigma^2 (\tau - t) \sigma \sqrt{\tau - t}.
\]
Pricing Call Options on Forwards

**Theorem 4**

*Suppose that $\tau \leq T$. Then it holds that*

$$\sup_{x \geq 0} |C(t, \tau, T, x) - C_{B76}(t, \tau, T, x)| \leq ((c_{3.6} + c_{3.7})x + (c_{3.4} + c_{3.5})K) e^{-\beta(T-\tau)},$$

*for constants*

$$c_{3.4} = \frac{1}{\sqrt{2\pi} \sigma} \left( \phi''(0) + \frac{1}{\beta} (\phi'(0))^2 \right),$$

$$c_{3.5} = \frac{1}{\sqrt{2\pi} \beta \sigma} \left( \int_{|z|<1} z^2 \ell(dz) + \int_{|z|\geq1} e^{2|z|} \ell(dz) \right),$$

$$c_{3.6} = \frac{1}{\sqrt{2\pi} \sigma} \left( \frac{1}{2} \sqrt{\beta} + \frac{1}{\sqrt{\beta}} \left( e^1 \int_{|z|<1} z^2 \ell(dz) + \int_{|z|\geq1} e^{3|z|} \ell(dz) \right) \right),$$

$$c_{3.7} = c_{3.5}.$$
Forward - Delivery Period

- Typically: forwards delivering the underlying energy over a delivery period $[T_1, T_2]$
- Forward price defined as expected average spot over the delivery period: no analytical closed form solution when the spot model is exponential
- Choose as delivery time some point in the delivery period, e.g. mid-point $T = (T_1 + T_2)/2$
Pricing Call Options on Forwards

- electricity markets: many options have exercise time equal to the beginning of delivery $T_1$ of the underlying forward, e.g. $\tau = T_1$.
- relatively long delivery period: $\tau$ is relatively far from $T^*$. 
- for a reasonably strong mean reversion $\beta$ of the spikes, options on forwards in electricity markets can be priced with a high degree of accuracy by the Black-76 formula.
Numerical example - Logarithmic pricing error

Figure: Difference of the option price to the Black-76 on log-scale. The solid line is a fitted line with slope $-\beta$. 
Sensitivity with respect to $\beta$

Figure: Relative pricing error in % as a function of the half life of the spike component with 5 spikes on average during a month. $T = 25$
Quadratic Hedging of Call Options on Forwards

- determine the quadratic hedging strategy for our market model
- quadratic hedge converges uniformly to the simple delta-hedging strategy
- rate of convergence same as for the price
Conclusion

- Based on a two-factor spot price model we show that
  - option prices converge exponentially to the Black-76 price in terms of
    - the speed of mean-reversion of the stationary factor in the spot price
    - the time left to maturity of the forward from the exercise time of the call.

- Combining
  - Approximation the delivery period by its midpoint
  - typically high speed of mean-reversion
  call options on electricity and gas forwards may be priced reasonably accurately by the Black-76 formula in many cases.

- Analogous results for quadratic hedging
How many mean-reverting components?

**Figure:** Empirical and fitted autocorrelation function of a filtered $Y$ in an arithmetic model. $\beta = 0.359$
Empirical Analysis - Illiquidity Issue

Figure: Phelix Option, EEX
Thank you for your attention!
Literature


