Modeling Trades in the Life Market as Nash Bargaining Problems: Methodology & Insights

Ken Seng Tan
kstan@uwaterloo.ca

Joint work with Rui Zhou and Johnny Li

8th Conference in Actuarial Science & Finance on Samos
May 29 - June 1, 2014
Presentation Outline

- Longevity
  - Evidence
  - Impact
  - Consequences

- De-Risking Solutions
  - A Tâtonnement Pricing Approach
    - Zhou, Li and Tan, forthcoming in JRI

- Nash Bargaining Solution
  - Zhou, Li and Tan (2013)
Life Expectancy At Birth

Males

Females

- Canada
- United States
- England & Wales

Sources: Kinsella and He (2009); United Nations (2011); and IMF staff estimates.
Note: The regional groupings reflect the UN classification.
Why is Longevity a “Risk”?

- Due to unexpected increases in life spans.
- IMF Global Financial Stability Report (April 2012): If individuals live three years longer than expected
  - Liabilities of corporate pension plans in the U.S. would rise by about 9%
  - The aggregate post-retirement living expenses in developed economies would increase by 50% of 2012 GDP.
- SOA’s "Key Finding and Issues: 2011 Risk and Process of Retirement Survey Report" emphasized that "[i]mproving the general public’s understanding of longevity and what it means for financial planning should be a high priority for all those committed to ensuring a secure retirement for American seniors."
- See also the article "Most Retirees Misjudge Life Expectancy" on July 30, 2012 by The Wall Street Journal (Smart Money).
Why is Longevity a “Risk”?

- Due to unexpected increases in life spans.
- IMF Global Financial Stability Report (April 2012): If individuals live three years longer than expected
  - Liabilities of corporate pension plans in the U.S. would rise by about 9%
  - The aggregate post-retirement living expenses in developed economies would increase by 50% of 2012 GDP.
- SOA’s "Key Finding and Issues: 2011 Risk and Process of Retirement Survey Report" emphasized that "[i]mproving the general public’s understanding of longevity and what it means for financial planning should be a high priority for all those committed to ensuring a secure retirement for American seniors."
- See also the article "Most Retirees Misjudge Life Expectancy" on July 30, 2012 by The Wall Street Journal (Smart Money).
Three-Pronged Approach Recommended by IMF

- “First, government should acknowledge the significant longevity risk they face through defined benefit plans for their employees and through old-age social security schemes.”

- “Second, risk should be appropriately shared between individuals, pension plan sponsors, and the government. An essential reform measure would allow retirement ages to increase along with expected longevity. This could be mandated by government, but individuals could also be encouraged to delay retirement voluntarily. Better education about longevity and its financial impact would help make the consequence clearer.”

- “Risk transfers in capital markets from pension plan to those that are better able to manage the risk are a third approach.”
De-Risking Solutions

Market-Based Transfer of Longevity Risk

- Pension buy-out
- Pension buy-in
- **Mortality-linked securities (MLS),** which are derivative securities with payoffs link to certain mortality or longevity indices
  - Longevity swap
  - Longevity bond
  - Mortality bond
## Some Recent Transactions

<table>
<thead>
<tr>
<th>Date</th>
<th>Hedger</th>
<th>Provider</th>
<th>Size</th>
<th>Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>May 14</td>
<td>Royal London</td>
<td>RGA Re</td>
<td>£1bn</td>
<td>longevity re.</td>
</tr>
<tr>
<td>March 14</td>
<td>AkzoNobel’s ICI</td>
<td>Legal &amp; General and Prudential</td>
<td>£1.5bn</td>
<td>pension buy-in</td>
</tr>
<tr>
<td>March 14</td>
<td>Aviva</td>
<td>Swiss Re &amp; SCOR</td>
<td>£5bn</td>
<td>longevity re.</td>
</tr>
<tr>
<td>Dec. 13</td>
<td>Aegon</td>
<td>SCOR</td>
<td>Euro 1.4bn</td>
<td>longevity swap</td>
</tr>
<tr>
<td>July 13</td>
<td>EMI Group</td>
<td>Pension Insurance Corporation</td>
<td>£1.5bn</td>
<td>pension buyout</td>
</tr>
<tr>
<td>June 13</td>
<td>Canadian Wheat Board</td>
<td>Sunlife</td>
<td>Cad$ 150m</td>
<td>pension buy-in</td>
</tr>
<tr>
<td>May 13</td>
<td>Bentley</td>
<td>Abbey Life</td>
<td>£400m</td>
<td>longevity swap</td>
</tr>
<tr>
<td>April 13</td>
<td>Abbey Life/ Rothesay Life</td>
<td>Hannover Re</td>
<td>£1bn</td>
<td>longevity re.</td>
</tr>
<tr>
<td>Dec. 12</td>
<td>LV=</td>
<td>Swiss Re</td>
<td>£3.2bn</td>
<td>longevity swap reinsurance</td>
</tr>
<tr>
<td>Feb. 13</td>
<td>BAE Systems</td>
<td>Legal &amp; General</td>
<td>£800m</td>
<td>longevity swap pensioner &amp; all members over age 55</td>
</tr>
<tr>
<td>June 12</td>
<td>GM</td>
<td>Prudential</td>
<td>US$ 26bn</td>
<td>pension buy-out</td>
</tr>
</tbody>
</table>
Hedging with Longevity Bond

An annuity provider or a pension sponsor

- Liability payments $f_t(Q^L_t)$ due at $t = 1, 2, \ldots, T$, where $Q^L_t$ is an index that contains information about the mortality of the population associated with Agent A’s annuity or pension liability.

Mortality-Linked Securities

- Longevity bond (MLS) with payout $g_t(Q^H_t)$ $t = 1, 2, \ldots, T$, where $Q^H_t$ is an index that contains information about the mortality of the population associated with the security.

- Both $Q^L_t$ and $Q^H_t$ are not necessarily identical.

- At time 0, the values of $Q^L_t$ and $Q^H_t$ for $t > 0$ are not known and are governed by some underlying stochastic processes.
Hedging with Longevity Bond

An annuity provider or a pension sponsor

- Liability payments $f_t(Q^L_t)$ due at $t = 1, 2, ..., T$, where $Q^L_t$ is an index that contains information about the mortality of the population associated with Agent A’s annuity or pension liability.

Mortality-Linked Securities

- Longevity bond (MLS) with payout $g_t(Q^H_t)$ $t = 1, 2, ..., T$, where $Q^H_t$ is an index that contains information about the mortality of the population associated with the security

- Both $Q^L_t$ and $Q^H_t$ are not necessary identical
- At time 0, the values of $Q^L_t$ and $Q^H_t$ for $t > 0$ are not known and are governed by some underlying stochastic processes.
Hedging with Longevity Bond

An annuity provider or a pension sponsor

- Liability payments $f_t(Q_t^L)$ due at $t = 1, 2, \ldots, T$, where $Q_t^L$ is an index that contains information about the mortality of the population associated with Agent A’s annuity or pension liability.

Mortality-Linked Securities

- Longevity bond (MLS) with payout $g_t(Q_t^H)$ $t = 1, 2, \ldots, T$, where $Q_t^H$ is an index that contains information about the mortality of the population associated with the security

- Both $Q_t^L$ and $Q_t^H$ are not necessarily identical

- At time 0, the values of $Q_t^L$ and $Q_t^H$ for $t > 0$ are not known and are governed by some underlying stochastic processes.
Hedging with Longevity Bond (cont’d)

Agent A

- An annuity provider or a pension sponsor who hedges her longevity exposure by buying longevity bond
- $\theta$ units of longevity bond

Agent B

- Issuer/seller of longevity bond
- Each unit of longevity bond is $P$

\[ f_t(Q_t^L) \]
\[ \theta g_t(Q_t^H) \]
\[ \theta P \]
Wealth Processes

Agent A:

\[ W_1^A(P, \theta) = (W_0^A - \theta P)e^r + \theta g_1(Q_{1}^H) - f_1(Q_{1}^L) \]
\[ W_t^A(P, \theta) = W_{t-1}^Ae^r + \theta g_t(Q_t^H) - f_t(Q_t^L), \quad t = 2, \ldots, T \]
\[ \Rightarrow W_T^A(P, \theta) = W_0^Ae^{rT} + \theta(G - Pe^{rT}) - F \]

- \( W_0^A \) is Agent A’s initial wealth and \( F = \sum_{t=1}^{T} f_t(Q_t)e^{r(T-t)} \).

Agent B:

\[ W_1^B(P, \theta) = (W_0^B + \theta P)e^r - \theta g_1(Q_{1}^H) \]
\[ W_t^B(P, \theta) = W_{t-1}^Be^r - \theta g_t(Q_t^H), \quad t = 2, \ldots, T \]
\[ \Rightarrow W_T^B(P, \theta) = W_0^Be^{rT} - \theta(G - Pe^{rT}) \]

- \( W_0^B \) is Agent B’s initial wealth and \( G = \sum_{t=1}^{T} g_t(Q_t)e^{r(T-t)} \).
Wealth Processes

Agent A:

\[
W^A_1(P, \theta) = (W^A_0 - \theta P)e^r + \theta g_1(Q^H_1) - f_1(Q^L_1)
\]

\[
W^A_t(P, \theta) = W^A_{t-1}e^r + \theta g_t(Q^H_t) - f_t(Q^L_t), \quad t = 2, \ldots, T
\]

\[
\Rightarrow W^A_T(P, \theta) = W^A_0e^{rT} + \theta (G - Pe^{rT}) - F
\]

- \(W^A_0\) is Agent A’s initial wealth and \(F = \sum_{t=1}^{T} f_t(Q_t)e^{r(T-t)}\).

Agent B:

\[
W^B_1(P, \theta) = (W^B_0 + \theta P)e^r - \theta g_1(Q^H_1)
\]

\[
W^B_t(P, \theta) = W^B_{t-1}e^r - \theta g_t(Q^H_t), \quad t = 2, \ldots, T
\]

\[
\Rightarrow W^B_T(P, \theta) = W^B_0e^{rT} - \theta (G - Pe^{rT})
\]

- \(W^B_0\) is Agent B’s initial wealth and \(G = \sum_{t=1}^{T} g_t(Q_t)e^{r(T-t)}\).
How to Price such MLS?

No-arbitrage Approach

- Complications?

Other Challenges

- multi-population mortality models
- basis risk
How to Price such MLS?

No-arbitrage Approach

- Complications?

Other Challenges

- multi-population mortality models
- basis risk

Our Proposed Two Approaches

Modeling Trade in a Competitive Market

• Based on Tâtonnement economic pricing method

Modeling Trade in a Non-Competitive Market

• Via Nash’s bargaining solution

• Borch (1974), Kihlstrom and Roth (1982), Schlesinger (1984), Boonen et al. (2012)
Our Proposed Two Approaches

Modeling Trade in a Competitive Market

- Based on Tâtonnement economic pricing method

Modeling Trade in a Non-Competitive Market

- Via Nash’s bargaining solution

A Tâtonnement Approach

- Agent A: buyer, hedger who exposes to longevity risk
- Agent B: selling MLS
- Auctioneer adjusts the price to match supply and demand
- Price adjustment stops when supply equals demand
Tâtonnement Approach (cont’d)

Maximizing expected terminal utilities

Agent A: \( \theta^A = \operatorname{argmax}_\theta \mathbb{E} \left[ U^A(W^A_T(P, \theta)) \right] \)

Agent B: \( \theta^B = \operatorname{argmax}_\theta \mathbb{E} \left[ U^B(W^B_T(P, \theta)) \right] \)

where \( U^A \) and \( U^B \) are the utility functions of Agents A and B, respectively.

The equilibrium Tâtonnement price \( P^* \)

\( \theta^A = \theta^B = \theta^* \)
Tâtonnement Approach (cont’d)

Maximizing expected terminal utilities

Agent A: \( \theta^A = \arg\max_\theta \mathbb{E}[U^A(W^A_T(P, \theta))] \)

Agent B: \( \theta^B = \arg\max_\theta \mathbb{E}[U^B(W^B_T(P, \theta))] \)

where \( U^A \) and \( U^B \) are the utility functions of Agents A and B, respectively.

The equilibrium Tâtonnement price \( P^* \)

\( \theta^A = \theta^B = \theta^* \)
Possible Scenarios

- Supply
- Demand

Price vs. Quantity Graphs for Different Scenarios
A Mortality Bond Example

- \( T = 3 \) years
- Life insurance benefits \( f_t(Q_t) = 1000q_t \)
- Risk exposure: \textit{mortality risk}

Hedging strategy:
- Agent A: Life insurer, issues mortality bond
- Agent B: Investor, buys mortality bond

Assume that the life insurance benefits and the payout from the mortality bond depend on the same underlying population
A Mortality Bond Example

- $T = 3$ years
- Life insurance benefits $f_t(Q_t) = 1000q_t$

- Risk exposure: *mortality risk*

- Hedging strategy:
  - Agent A: Life insurer, issues mortality bond
  - Agent B: Investor, buys mortality bond

- Assume that the life insurance benefits and the payout from the mortality bond depend on the same underlying population
A Mortality Bond Example (cont’d)

- A mortality bond with 3 years maturity
- Face value $1
- $r = 3\%$
- Annual coupon rate = $r + 1.5\%$
- Principal Repayment = $\max \left( 1 - \sum_{t=1}^{3} \text{loss}_t, 0 \right)$, where

\[
\text{loss}_t = \frac{\max(q_t - 1.1q_0, 0) - \max(q_t - 1.2q_0, 0)}{0.1q_0}
\]

- Similar to the mortality bond issued by Swiss Re in December 2003
A Mortality Bond Example (cont’d)

Utility functions

- Both agents have exponential utility functions

\[ U(x) = 1 - e^{-kx} \]

- Set \( k^A = 1.0, \ k^B = 0.5 \) (see Emms and Haberman, 2009)

Stochastic mortality model

- A generalization of Lee and Carter (1992) model, which incorporates jumps, proposed by Chen and Cox (2009)
Supply and Demand Curves

Tâtonnement Approach

- Equilibrium price $P^* = 1.0319$
- $|\theta^A| = |\theta^B| = \theta^* = 1.86$ units.
Sensitivity of Pricing Methods

Chen and Cox (2009)’s Pricing Method

- Use Wang’s transform to identify a risk-neutral probability measure
- Then calculate the price of the security
- A potential issue: need to calibrate “market price of risks” $\lambda_1, \lambda_2, \lambda_3$

Chen and Cox’s results (compare with $P^* = $1.0319)

<table>
<thead>
<tr>
<th>$\lambda_1$</th>
<th>5.1449</th>
<th>0</th>
<th>1.500</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_2$</td>
<td>0</td>
<td>3.4808</td>
<td>1.500</td>
</tr>
<tr>
<td>$\lambda_3$</td>
<td>0</td>
<td>0</td>
<td>1.500</td>
</tr>
</tbody>
</table>

Price

$\$0.1318$ $\$0.9384$ $\$0.4976$
Tâtonnement Approach: Some Results

- Under the assumption of exponential utilities, we establish
  - $P^*$ is independent of the initial wealths $W^0_A$ and $W^0_B$.
  - $P^*$ exists and unique (under some condition)
  - The competitive equilibrium $(P^*, \theta^*)$ satisfies

$$P^* = \frac{\mathbb{E}[e^{-k^A\theta^* G+k^AF} G]}{e^{rT}\mathbb{E}[e^{-k^A\theta^* G+k^AF}]} = \frac{\mathbb{E}[e^{k^B\theta^* G} G]}{e^{rT}\mathbb{E}[e^{k^B\theta^* G}]}.$$  

where $k^A$ and $k^B$ are the absolute risk aversion parameters of Agents A and B, respectively, and recall

$$F = \sum_{t=1}^{T} f_t(Q_t)e^{r(T-t)}, \quad G = \sum_{t=1}^{T} g_t(Q_t)e^{r(T-t)}$$

- Extended to dynamic trading strategy
Tâtonnement Approach: Some Results

- Under the assumption of exponential utilities, we establish
  - $P^*$ is independent of the initial wealths $W^0_A$ and $W^0_B$.
  - $P^*$ exists and unique (under some condition)
  - The competitive equilibrium $(P^*, \theta^*)$ satisfies

$$P^* = \frac{\mathbb{E}[e^{-k^A\theta^* G + k^AF}G]}{e^{rT} \mathbb{E}[e^{-k^A\theta^* G + k^AF}]} = \frac{\mathbb{E}[e^{k^B\theta^* G}G]}{e^{rT} \mathbb{E}[e^{k^B\theta^* G}]}.$$ 

where $k^A$ and $k^B$ are the absolute risk aversion parameters of Agents A and B, respectively, and recall

$$F = \sum_{t=1}^{T} f_t(Q_t) e^{r(T-t)}, \quad G = \sum_{t=1}^{T} g_t(Q_t) e^{r(T-t)}$$

- Extended to dynamic trading strategy
Zhou, Li and Tan (2013)

Modeling Trade in a Non-Competitive Market

- Formulate as a Nash bargaining problem (1950) with two agents
- A Nash bargaining problem is a pair $\langle S, d \rangle$, where $S \subset \mathbb{R}^2$ is a compact and convex set, $d = (d_1, d_2) \in S$, and for some $s = (s_1, s_2) \in S$, $s_i > d_i$ for $i = 1, 2$.
  - $S$ is the set of all feasible expected utility payoffs to the agents,
  - $d = (d_1, d_2)$ represents the disagreement payoff;
  - if the agents do not come to an agreement, then they will receive utility payoffs of $d_1$ and $d_2$, respectively.
- If there exists $s = (s_1, s_2) \in S$ such that $s_i > d_i$ for $i = 1, 2$, then the agents have incentive to reach an agreement.
Nash Bargaining Problem

- The set of all bargaining problems is denoted by $B$.

- A **bargaining solution** is a function $\zeta : B \rightarrow \mathbb{R}^2$ that assigns to each bargaining problem $\langle S, d \rangle \in B$ a unique element of $S$. 
Nash’s Axioms

1. **Pareto optimality**
   If \( \zeta(S, d) = (z_1, z_2) \) and \( y_i \geq z_i \) for \( i = 1, 2 \), then either \( y_i = z_i \) for \( i = 1, 2 \) or \( (y_1, y_2) \notin S \).

2. **Independence of equivalent utility representatives**
   If \( (S', d') \) is related to \( (S, d) \) in such a way that \( d'_i = a_i d_i + b_i \) and \( s'_i = a_i s_i + b_i \) for \( i = 1, 2 \), where \( a_i \) and \( b_i \) are real numbers and \( a_i > 0 \), then
   \[
   \zeta_i(S', d') = a_i \zeta_i(S, d) + b_i \quad \text{for} \quad i = 1, 2.
   \]

3. **Symmetry**
   - The bargaining problem \( (S, d) \) is **symmetric** if \( d_1 = d_2 \) and \( (x_1, x_2) \in S \) iff \( (x_2, x_1) \in S \).
   - If the bargaining problem \( (S, d) \) is symmetric, then \( \zeta_1(S, d) = \zeta_2(S, d) \).

4. **Independence of irrelevant alternatives**
   If \( (S, d) \) and \( (T, d) \) are bargaining problems such that \( S \subset T \) and \( \zeta(T, d) \in S \), then \( \zeta(S, d) = \zeta(T, d) \).
Nash’s Theorem

- There is a unique solution which possesses Axioms 1-4. The solution, $\zeta^N(S, d) : B \rightarrow \mathbb{R}^2$, takes the form

$$\zeta^N(S, d) = \arg \max (s_1 - d_1)(s_2 - d_2),$$

where the maximization is taken over $(s_1, s_2) \in S$, and is subject to the constraint $s_i > d_i$ for $i = 1, 2$.

- $(s_1 - d_1)(s_2 - d_2)$ is known as the Nash product.
Nash’s Bargaining Problem and MLS Pricing

- The utility possible set $S$ is the set of feasible expected utility pairs

$$
\left( \mathbb{E} \left[ U^A(W^A_T(P, \theta)) \right], \mathbb{E} \left[ U^B(W^B_T(P, \theta)) \right] \right)
$$

arising from all possible values of $P$ and $\theta$.

- The agents are only allowed to bargain over the price $P$ and the quantity $\theta$.

- The disagreement utility payoffs are the expected terminal utilities when there is no trade (i.e., $\theta = 0$); i.e.,

$$
d = \left( \mathbb{E} \left[ U^A(W^A_T(0, 0)) \right], \mathbb{E} \left[ U^B(W^B_T(0, 0)) \right] \right).
$$

It is obvious that $d \in S$. 
Nash’s Bargaining Solution for the Price of MLS

\[
\text{argmax}_{(P, \theta)} \left( \mathbb{E} \left[ U^A(W^A_T(P, \theta)) \right] - \mathbb{E} \left[ U^A(W^A_T(0, 0)) \right] \right) \\
\times \left( \mathbb{E} \left[ U^B(W^B_T(P, \theta)) \right] - \mathbb{E} \left[ U^B(W^B_T(0, 0)) \right] \right)
\]

subject to

\[
\mathbb{E} \left[ U^A(W^A_T(P, \theta)) \right] - \mathbb{E} \left[ U^A(W^A_T(0, 0)) \right] \geq 0
\]

\[
\mathbb{E} \left[ U^B(W^B_T(P, \theta)) \right] - \mathbb{E} \left[ U^B(W^B_T(0, 0)) \right] \geq 0
\]

\[
\theta \geq 0
\]

\[
P > 0
\]
A Longevity Bond Example

Agent A

- Agent A: A pension plan sponsor who manages a closed pension plan with 1,500 pensioners
- Each pensioner receives $0.01 at the end each year until he reaches age 90 or dies
- Assume that the plan members’ future mortality experience is the same as that of the U.K. insured lives
A Longevity Bond Example (cont’d)

Agent B

- An investment bank who issues a 25-year annuity bond with payoff ties to the survivorship of English and Welsh male population; i.e.

\[ Q^H_t = \prod_{i=1}^{t} (1 - m^{(1)}_{64+t,2005+t}) \]

Other Assumptions

- The continuously compounded risk-free interest rate is \( r = 0.01 \).
- \( k^A = 2.0 \) and \( k^B = 0.1 \)
- Two-population model of Cairns et al (2011) is calibrated to the mortality data
A Longevity Bond Example (cont’d)

Agent B

- An investment bank who issues a 25-year annuity bond with payoff ties to the survivorship of English and Welsh male population; i.e.

\[ Q_t^H = \prod_{i=1}^{t} (1 - m_{i}^{(1)}_{64+t,2005+t}) \]

Other Assumptions

- The continuously compounded risk-free interest rate is \( r = 0.01 \).
- \( k^A = 2.0 \) and \( k^B = 0.1 \)
- Two-population model of Cairns et al (2011) is calibrated to the mortality data
## Results: Competitive vs Nash

<table>
<thead>
<tr>
<th>Method</th>
<th>Competitive Equilibrium</th>
<th>Nash’s Bargaining Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trading Price</td>
<td>15.6184</td>
<td>16.2410</td>
</tr>
<tr>
<td>Utility Gain for A</td>
<td>3.4804</td>
<td>3.2627</td>
</tr>
<tr>
<td>Utility Gain for B</td>
<td>0.0614</td>
<td>0.4282</td>
</tr>
<tr>
<td>Nash Product</td>
<td>0.2137</td>
<td>1.3971</td>
</tr>
</tbody>
</table>
### Sensitivity wrt Risk Aversion

#### $k^A$ is fixed to 2.0, $k^B$ is varied

<table>
<thead>
<tr>
<th>$k^B$</th>
<th>Competitive Equilibrium</th>
<th>Nash’s Bargaining Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Price</td>
<td>Quantity</td>
</tr>
<tr>
<td>0.3</td>
<td>15.8956</td>
<td>5.8428</td>
</tr>
<tr>
<td>0.5</td>
<td>16.1221</td>
<td>5.4814</td>
</tr>
<tr>
<td>0.7</td>
<td>16.2929</td>
<td>5.0425</td>
</tr>
</tbody>
</table>

#### $k^A$ is varied, $k^B$ is fixed to 0.1

<table>
<thead>
<tr>
<th>$k^A$</th>
<th>Competitive Equilibrium</th>
<th>Nash’s Bargaining Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Price</td>
<td>Quantity</td>
</tr>
<tr>
<td>1.5</td>
<td>15.6100</td>
<td>5.8711</td>
</tr>
<tr>
<td>1.0</td>
<td>15.5993</td>
<td>5.4500</td>
</tr>
<tr>
<td>0.5</td>
<td>15.5882</td>
<td>5.0151</td>
</tr>
</tbody>
</table>
Additional Results

Pareto Optimality

Assume that Agents A and B have exponential utility functions with risk aversion parameters $k^A$ and $k^B$, respectively.

- When $\text{cov}(e^{k^A F}, G) \leq 0$, the outcome $(\tilde{P}, \tilde{\theta})$ is Pareto optimal if and only if $\tilde{\theta} = 0$.
- When $\text{cov}(e^{k^A F}, G) > 0$, the outcome $(\tilde{P}, \tilde{\theta})$ is Pareto optimal if and only if $\mathcal{H}(\tilde{\theta}) = 0$, where

$$
\mathcal{H}(\theta) = \frac{\mathbb{E}[e^{k^B \theta G} G]}{\mathbb{E}[e^{k^B \theta G}]} - \frac{\mathbb{E}[e^{-k^A \theta G + k^A F} G]}{\mathbb{E}[e^{-k^A \theta G + k^A F}]} = 0
$$

Moreover, the solution is unique.
Additional Results (cont’d)

Condition for $s_i > d_i$

- Assume that Agents A and B have exponential utility functions with risk aversion parameters $k^A$ and $k^B$, respectively.

- A necessary and sufficient condition for satisfying the assumption that there exists $s = (s_1, s_2)$ in $S$ such that

$$s_i > d_i$$

for $i = 1, 2$ is $\text{cov}(e^{k^AF}, G) > 0$. 
Concluding Remarks

- We presented two approaches for pricing MLS: Tâtonnement Method and Nash’s Bargaining Method.
- Both methods do not need market price data.
- Assuming the hedger and investor have exponential utility functions,
  - a trade would occur if the longevity security is an effective hedging instrument, in the sense that $\text{cov}(e^{kA_F}, G) > 0$.
  - provided that a trade occurs, the two set-ups would result in the same trading quantity but different trading prices.
- Other utility functions?

Thank You For Your Attention
Concluding Remarks

- We presented two approaches for pricing MLS: Tâtonnement Method and Nash’s Bargaining Method.
- Both methods do not need market price data.
- Assuming the hedger and investor have exponential utility functions,
  - a trade would occur if the longevity security is an effective hedging instrument, in the sense that $\text{cov}(e^{kA}F, G) > 0$.
  - provided that a trade occurs, the two set-ups would result in the same trading quantity but different trading prices.
- Other utility functions?

Thank You For Your Attention
Concluding Remarks

- We presented two approaches for pricing MLS: Tâtonnement Method and Nash’s Bargaining Method
- Both methods do not need market price data
- Assuming the hedger and investor have exponential utility functions,
  - a trade would occur if the longevity security is an effective hedging instrument, in the sense that $\text{cov}(e^{kF}, G) > 0$
  - provided that a trade occurs, the two set-ups would result in the same trading quantity but different trading prices
- Other utility functions?

Thank You For Your Attention