Modeling Trades in the Life Market as Nash Bargaining Problems: Methodology & Insights

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Joint work with Rui Zhou and Johnny Li

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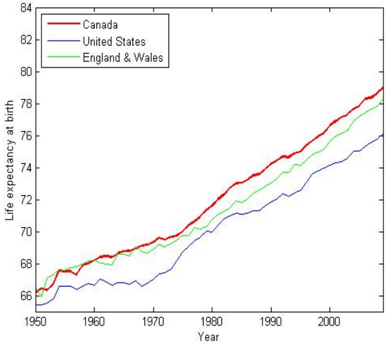
Presentation Outline

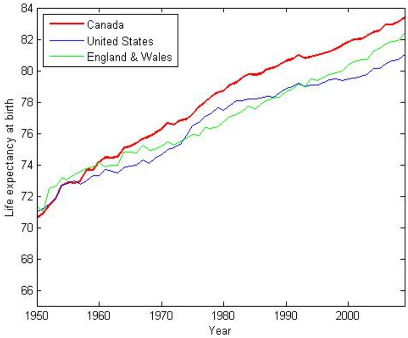
- Longevity
    - Evidence
    - Impact
    - Consequences
- De-Risking Solutions
- A Tâtonnement Pricing Approach
    - Zhou, Li and Tan, forthcoming in JRI
- Nash Bargaining Solution
    - Zhou, Li and Tan (2013)

Life Expectancy At Birth

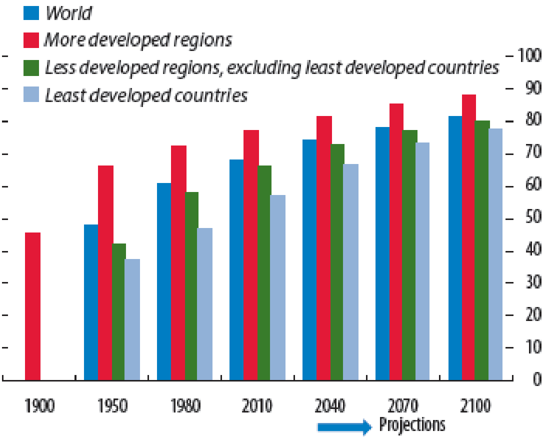
Males



Females



Life Expectancy at Birth: IMF Global Financial Stability Report 2012



Sources: Kinsella and He (2009); United Nations (2011); and IMF staff estimates.
Note: The regional groupings reflect the UN classification.

Why is Longevity a “Risk"?

- Due to **unexpected** increases in life spans.
- IMF Global Financial Stability Report (April 2012): If individuals live three years longer than expected
  - Liabilities of corporate pension plans in the U.S. would rise by about 9%
  - The aggregate post-retirement living expenses in developed economies would increase by 50% of 2012 GDP.
- SOA's "Key Finding and Issues: 2011 Risk and Process of Retirement Survey Report" emphasized that *"[i]mproving the general public's understanding of longevity and what it means for financial planning should be a high priority for all those committed to ensuring a secure retirement for American seniors."*
- See also the article "Most Retirees Misjudge Life Expectancy" on July 30, 2012 by The Wall Street Journal (Smart Money).

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Three-Pronged Approach Recommended by IMF

- *" First, government should acknowledge the significant longevity rik they face through defind benefit-plans for their employees and through old-age social security schemes."*
- *"Second, risk should be appropriately shared between individuals, pension plan sponsors, and the government. An essential reform measure would allow retirement ages to increase along with expected longevity. This could be mandated by government, but individuals could also be encouraged to delay retirement voluntarily. Better education about longevity and its financial impact would help make the consequence clearer."*
- *" Risk transfers in capital markets from pension plan to those that are better abe to manage the risk are a third approach."*

De-Risking Solutions

Market-Based Transfer of Longevity Risk

- Pension buy-out
- Pension buy-in
- Mortality-linked securities (MLS), which are derivative securities with payoffs link to certain mortality or longevity indices
  - Longevity swap
  - Longevity bond
  - Mortality bond

Some Recent Transactions

Date	Hedger	Provider	Size	Solution
May 14	Royal London	RGA Re	£1bn	longevity re.
March 14	AkzoNobel's ICI Pension Fund	Legal & General and Prudential	£1.5bn	pension buy-in
March 14	Aviva	Swiss Re & SCOR	£5bn	longevity re.
Dec. 13	Aegon	SCOR	Euro 1.4bn	longevity swap
July 13	EMI Group Pension Fund	Pension Insurance Corporation	£1.5bn	pension buyout
June 13	Canadian Wheat Board	Sunlife	Cad$ 150m	pension buy-in
May 13	Bentley	Abbey Life	£400m	longevity swap
April 13	Abbey Life/ Rothesay Life	Hannover Re	£1bn	longevity reinsurance
Feb. 13	BAE Systems	Legal & General	£3.2bn	longevity swap
Dec. 12	LV=	Swiss Re	£800m	pensioner & all members over age 55
June 12	GM	Prudential	US$ 26bn	pension buy-out

Hedging with Longevity Bond

An annuity provider or a pension sponsor

- Liability payments $f_t(Q_t^L)$ due at $t = 1, 2, ..., T$, where Q_t^L is an index that contains information about the mortality of the population associated with Agent A's annuity or pension liability.

Mortality-Linked Securities

- Longevity bond (MLS) with payout $g_t(Q_t^H)$ $t = 1, 2, ..., T$, where Q_t^H is an index that contains information about the mortality of the population associated with the security
- Both Q_t^L and Q_t^H are not necessary identical
- At time 0, the values of Q_t^L and Q_t^H for $t > 0$ are not known and are governed by some underlying stochastic processes.

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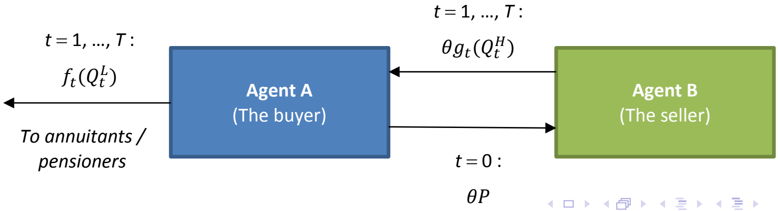
Hedging with Longevity Bond (cont'd)

Agent A

- An annuity provider or a pension sponsor who hedges her longevity exposure by buying longevity bond
- θ units of longevity bond

Agent B

- issuer/seller of longevity bond
- each unit of longevity bond is P



Wealth Processes

Agent A:

$$\begin{aligned} W_1^A(P,\theta) &= (W_0^A - \theta P)e^r + \theta g_1(Q_1^H) - f_1(Q_1^L) \\ W_t^A(P,\theta) &= W_{t-1}^A e^r + \theta g_t(Q_t^H) - f_t(Q_t^L), \quad t = 2, \dots, T \\ \Rightarrow W_T^A(P,\theta) &= W_0^A e^{rT} + \theta(G - Pe^{rT}) - F \end{aligned}$$

- W_0^A is Agent A's initial wealth and $F = \sum_{t=1}^T f_t(Q_t)e^{r(T-t)}$.

Agent B:

$$\begin{aligned} W_1^B(P,\theta) &= (W_0^B + \theta P)e^r - \theta g_1(Q_1^H) \\ W_t^B(P,\theta) &= W_{t-1}^B e^r - \theta g_t(Q_t^H), \quad t = 2, \dots, T \\ \Rightarrow W_T^B(P,\theta) &= W_0^B e^{rT} - \theta(G - Pe^{rT}) \end{aligned}$$

- W_0^B is Agent B's initial wealth and $G = \sum_{t=1}^T g_t(Q_t)e^{r(T-t)}$.

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- W_0^B is Agent B's initial wealth and $G = \sum_{t=1}^{T} g_t(Q_t)e^{r(T-t)}$.

How to Price such MLS?

No-arbitrage Approach

- Cairns, Blake and Dowd (2006), Lin and Cox (2005), Denuit, Devolder, and Goderniaux (2007), Dowd et al. (2006), Chen and Cox (2009), Wang and Yang (2013), ...
- Complications?

Other Challenges

- multi-population mortality models
- basis risk

⇒ Cairns, et al (2011), Lin, Liu and Yu (2013), Zhou, Li and Tan (2013), Millossovich, Danesi and Haberman (2014), ...

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Our Proposed Two Approaches

Modeling Trade in a Competitive Market

- Based on Tâtonnement economic pricing method

  ⇒ *Zhou, R., J.S.H. Li and K.S. Tan "Economic pricing of mortality-linked securities: a Tâtonnement approach," to appear in* Journal of Risk and Insurance.

Modeling Trade in a Non-Competitive Market

- Via Nash's bargaining solution

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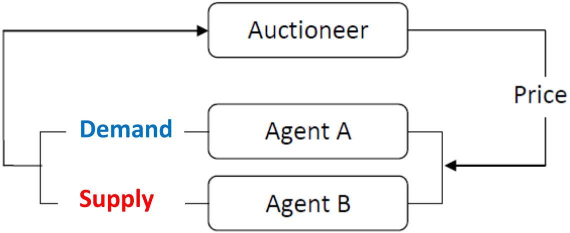
A Tâtonnement Approach



- Agent A: buyer, hedger who exposes to longevity risk
- Agent B: selling MLS
- Auctioneer adjusts the price to match supply and demand
- Price adjustment stops when supply equals demand

Tâtonnement Approach (cont'd)

Maximizing expected terminal utilities

$$\text{Agent A:} \quad \theta^A = \underset{\theta}{\operatorname{argmax}}\, \mathbb{E}\left[U^A(W_T^A(P,\theta))\right]$$

$$\text{Agent B:} \quad \theta^B = \underset{\theta}{\operatorname{argmax}}\, \mathbb{E}\left[U^B(W_T^B(P,\theta))\right]$$

where U^A and U^B are the utility functions of Agents A and B, respectively.

The equilibrium Tâtonnement price P^*

$$\theta^A = \theta^B = \theta^*$$

Tâtonnement Approach (cont'd)

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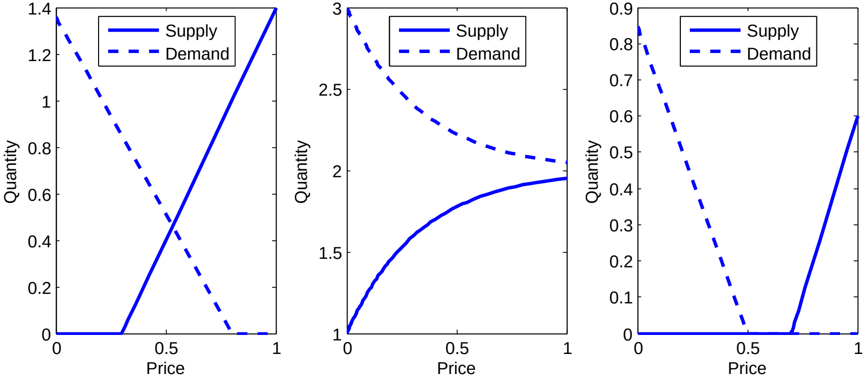
Possible Scenarios



A Mortality Bond Example

- $T = 3$ years
- Life insurance benefits $f_t(Q_t) = 1000q_t$

- Risk exposure: *mortality risk*

- Hedging strategy:
  - Agent A: Life insurer, issues mortality bond
  - Agent B: Investor, buys mortality bond

- Assume that the life insurance benefits and the payout from the mortality bond depend on the same underlying population

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A Mortality Bond Example (cont'd)

- A mortality bond with 3 years maturity
- Face value \$1
- $r = 3\%$
- Annual coupon rate = $r + 1.5\%$
- Principal Repayment $= \max\left(1 - \sum_{t=1}^{3} \text{loss}_t, 0\right)$, where

$$\text{loss}_t = \frac{\max(q_t - 1.1q_0, 0) - \max(q_t - 1.2q_0, 0)}{0.1q_0}$$

- Similar to the mortality bond issued by Swiss Re in December 2003

A Mortality Bond Example (cont'd)

Utility functions

- Both agents have exponential utility functions

$$U(x) = 1 - e^{-kx}$$

- Set $k^A = 1.0$, $k^B = 0.5$ (see Emms and Haberman, 2009)

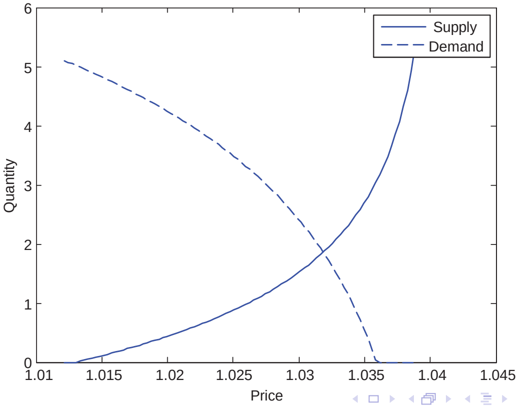
Stochastic mortality model

- A generalization of Lee and Carter (1992) model, which incorporates jumps, proposed by Chen and Cox (2009)

Supply and Demand Curves

Tâtonnement Approach

- Equilibrium price $P^* = \$1.0319$
- $|\theta^A| = |\theta^B| = \theta^* = 1.86$ units.



Sensitivity of Pricing Methods

Chen and Cox (2009)'s Pricing Method

- Use Wang's transform to identify a risk-neutral probability measure
- Then calculate the price of the security
- A potential issue: need to calibrate "market price of risks" $\lambda_1, \lambda_2, \lambda_3$

Chen and Cox's results (compare with $P^* = \$1.0319$)

λ_1	5.1449	0	1.500
λ_2	0	3.4808	1.500
λ_3	0	0	1.500
Price	\$0.1318	\$0.9384	\$0.4976

Tâtonnement Approach: Some Results

- Under the assumption of exponential utilities, we establish
  - P^* is independent of the initial wealths W_A^0 and W_0^B .
  - P^* exists and unique (under some condition)
  - The competitive equilibrium (P^*, θ^*) satisfies

$$P^* = \frac{\mathbb{E}[e^{-k^A\theta^* G + k^A F} G]}{e^{rT}\mathbb{E}[e^{-k^A\theta^* G + k^A F}]} = \frac{\mathbb{E}[e^{k^B\theta^* G} G]}{e^{rT}\mathbb{E}[e^{k^B\theta^* G}]}.$$

where k^A and k^B are the absolute risk aversion parameters of Agents A and B, respectively, and recall

$$F = \sum_{t=1}^{T} f_t(Q_t) e^{r(T-t)}, \qquad G = \sum_{t=1}^{T} g_t(Q_t) e^{r(T-t)}$$

- Extended to dynamic trading strategy

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- Extended to dynamic trading strategy

Zhou, Li and Tan (2013)

Modeling Trade in a Non-Competitive Market

- Formulate as a Nash bargaining problem (1950) with two agents
- A **Nash bargaining problem** is a pair $\langle S, d\rangle$, where $S \subset \Re^2$ is a compact and convex set, $d = (d_1, d_2) \in S$, and for some $s = (s_1, s_2) \in S$, $s_i > d_i$ for $i = 1, 2$.
  - S is the set of all feasible expected utility payoffs to the agents,
  - $d = (d_1, d_2)$ represents the disagreement payoff;
  - if the agents do not come to an agreement, then they will receive utility payoffs of d_1 and d_2, respectively.
- If there exists $s = (s_1, s_2)$ in S such that $s_i > d_i$ for $i = 1, 2$, then the agents have incentive to reach an agreement.

Nash Bargaining Problem

- The set of all bargaining problems is denoted by $\mathcal{B}$
- A **bargaining solution** is a function $\zeta : \mathcal{B} \rightarrow \Re^2$ that assigns to each bargaining problem $\langle S, d \rangle \in \mathcal{B}$ a unique element of S.

Nash's Axioms

1. **Pareto optimality**
   If $\zeta(S, d) = (z_1, z_2)$ and $y_i \geq z_i$ for $i = 1, 2$, then either $y_i = z_i$ for $i = 1, 2$ or $(y_1, y_2) \notin S$.
2. **Independence of equivalent utility representatives**
   If (S', d') is related to (S, d) in such a way that $d'_i = a_i d_i + b_i$ and $s'_i = a_i s_i + b_i$ for $i = 1, 2$, where a_i and b_i are real numbers and $a_i > 0$, then
   $\zeta_i(S', d') = a_i \zeta_i(S, d) + b_i$ for $i = 1, 2$.
3. **Symmetry**
   - The bargaining problem (S, d) is **symmetric** if $d_1 = d_2$ and $(x_1, x_2) \in S$ iff $(x_2, x_1) \in S$.
   - If the bargaining problem (S, d) is symmetric, then $\zeta_1(S, d) = \zeta_2(S, d)$.
4. **Independence of irrelevant alternatives**
   If (S, d) and (T, d) are bargaining problems such that $S \subset T$ and $\zeta(T, d) \in S$, then $\zeta(S, d) = \zeta(T, d)$.

Nash's Theorem

- There is a unique solution which possesses Axioms 1-4. The solution, $\zeta^N(S, d) : \mathcal{B} \to \Re^2$, takes the form

$$\zeta^N(S, d) = \arg\max(s_1 - d_1)(s_2 - d_2),$$

where the maximization is taken over $(s_1, s_2) \in S$, and is subject to the constraint $s_i > d_i$ for $i = 1, 2$.

- $(s_1 - d_1)(s_2 - d_2)$ is known as the **Nash product**.

Nash's Bargaining Problem and MLS Pricing

- The utility possible set S is the set of feasible expected utility pairs

$$\left(\mathbb{E}\left[U^A(W_T^A(P,\theta))\right],\mathbb{E}\left[U^B(W_T^B(P,\theta))\right]\right)$$

arising from all possible values of P and θ.

- The agents are only allowed to bargain over the price P and the quantity θ.
- The disagreement utility payoffs are the expected terminal utilities when there is no trade (i.e., $\theta = 0$); i.e.,

$$d = \left(\mathbb{E}\left[U^A(W_T^A(0,0))\right],\mathbb{E}\left[U^B(W_T^B(0,0))\right]\right).$$

It is obvious that $d \in S$.

Nash's Bargaining Solution for the Price of MLS

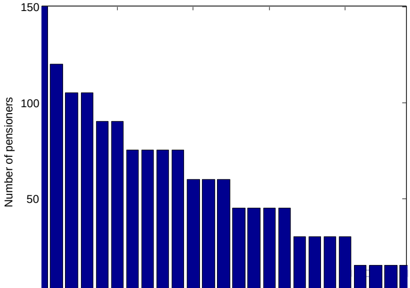
$$\underset{(P,\theta)}{\text{argmax}} \quad \left(\mathbb{E}\left[U^A(W_T^A(P,\theta))\right] - \mathbb{E}\left[U^A(W_T^A(0,0))\right]\right)$$
$$\times \left(\mathbb{E}\left[U^B(W_T^B(P,\theta))\right] - \mathbb{E}\left[U^B(W_T^B(0,0))\right]\right)$$

$$\text{subject to} \quad \mathbb{E}\left[U^A(W_T^A(P,\theta))\right] - \mathbb{E}\left[U^A(W_T^A(0,0))\right] \geq 0$$
$$\mathbb{E}\left[U^B(W_T^B(P,\theta))\right] - \mathbb{E}\left[U^B(W_T^B(0,0))\right] \geq 0$$
$$\theta \geq 0$$
$$P > 0$$

A Longevity Bond Example

Agent A

- Agent A: A pension plan sponsor who manages a closed pension plan with 1,500 pensioners
- Each pensioner receives $0.01 at the end each year until he reaches age 90 or dies
- Assume that the plan members' future mortality experience is the same as that of the U.K. insured lives



A Longevity Bond Example (cont'd)

Agent B

- An investment bank who issues a 25-year annuity bond with payoff ties to the survivorship of English and Welsh male population; i.e.

$$Q_t^H = \prod_{i=1}^{t}(1 - m_{64+t,2005+t}^{(1)})$$

Other Assumptions

- The continuously compounded risk-free interest rate is $r = 0.01$.
- $k^A = 2.0$ and $k^B = 0.1$
- Two-population model of Cairns et al (2011) is calibrated to the mortality data

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Results: Competitive vs Nash

Method	Competitive Equilibrium	Nash's Bargaining Solution
Trading Price	15.6184	16.2410
Trading Quantity	6.1997	6.1997
Utility Gain for A	3.4804	3.2627
Utility Gain for B	0.0614	0.4282
Nash Product	0.2137	1.3971

Sensitivity wrt Risk Aversion

k^A is fixed to 2.0, k^B is varied				
	Competitive Equilibrium		Nash's Bargaining Solution	
k^B	Price	Quantity	Price	Quantity
0.1	15.6184	6.1997	16.2410	6.1997
0.3	15.8956	5.8428	16.2594	5.8428
0.5	16.1221	5.4814	16.2770	5.4814
0.7	16.2929	5.0425	16.2939	5.0425

k^A is varied, k^B is fixed to 0.1				
	Competitive Equilibrium		Nash's Bargaining Solution	
k^A	Price	Quantity	Price	Quantity
2.0	15.6184	6.1997	16.2410	6.1997
1.5	15.6100	5.8711	16.1632	5.8711
1.0	15.5993	5.4500	15.5985	5.4500
0.5	15.5882	5.0151	15.7261	5.0151

Additional Results

Pareto Optimality

Assume that Agents A and B have exponential utility functions with risk aversion parameters k^A and k^B, respectively.

- When $cov(e^{k^A F}, G) \leq 0$, the outcome $(\tilde{P}, \tilde{\theta})$ is Pareto optimal if and only if $\tilde{\theta} = 0$.
- When $cov(e^{k^A F}, G) > 0$, the outcome $(\tilde{P}, \tilde{\theta})$ is Pareto optimal if and only if $\mathcal{H}(\tilde{\theta}) = 0$, where

$$\mathcal{H}(\theta) = \frac{\mathbb{E}[e^{k^B \theta G} G]}{\mathbb{E}[e^{k^B \theta G}]} - \frac{\mathbb{E}[e^{-k^A \theta G + k^A F} G]}{\mathbb{E}[e^{-k^A \theta G + k^A F}]} = 0$$

Moreover, the solution is unique.

Additional Results (cont'd)

Condition for $s_i > d_i$

- Assume that Agents A and B have exponential utility functions with risk aversion parameters k^A and k^B, respectively.
- A necessary and sufficient condition for satisfying the assumption that there exists $s = (s_1, s_2)$ in S such that

$$s_i > d_i$$

for $i = 1, 2$ is $\text{cov}(e^{k^A F}, G) > 0$.

Concluding Remarks

- We presented two approaches for pricing MLS: Tâtonnement Method and Nash's Bargaining Method
- Both methods do not need market price data
- Assuming the hedger and investor have exponential utility functions,
  - a trade would occur if the longevity security is an effective hedging instrument, in the sense that $\text{cov}(e^{k^A F}, G) > 0$
  - provided that a trade occurs, the two set-ups would result in the same trading quantity but different trading prices
- Other utility functions?

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